Teaching about Women and Mathematics:
Materials and Resources for an Interdisciplinary Course

Dr. Jacqueline Dewar, Dr. Lily Khadjavi and Dr. Alissa Crans

Loyola Marymount University
Los Angeles, CA

NOTE TO OUR READERS:

When a course such as this is developed and revised over time, materials are drawn from variety of sources and experiences. To the best of our ability, we have cited or acknowledged those sources. We ask that the reader do the same if she or he uses materials from this manual.

PREFACE

This manual provides information and materials for teaching a college-level course on women and mathematics that addresses both the lives and work of women mathematicians past and current, and examines gender-related concerns about women's participation in mathematics from kindergarten through graduate school and in math-related careers.

The purpose of this manual is two-fold:

- To make available information and materials used in an interdisciplinary course addressing women and mathematics at Loyola Marymount University (LMU) in Los Angeles, CA;
- To document the results of the Tensor-MAA grant (2007-2011), Women and Mathematics for Future Teachers, which funded the team-teaching of this course at LMU, by its original developer, Dr. Jacqueline Dewar, and two junior faculty members, Dr. Lily Khadjavi and Dr. Alissa Crans.

We wish to acknowledge the importance of the Tensor-MAA funding in supporting our team teaching and our efforts to gain recognition for the value of teaching a course like this, and for encouraging us to make these materials available to a broader audience through this manual. We are particularly grateful to Dr. Florence Fasanelli, former director of the Tensor-MAA Women and Mathematics program, for her support and encouragement throughout two rounds of funding.

For more information, or if you have suggestions or corrections, contact:

jdewar@lmu.edu

CONTENTS

2.3. "Women & Mathematics" for Future Teachers: Outcomes of a Tensor-MAA Women and Mathematics Grant, poster by A. Crans, J.Dewar and L. Khadjavi, Mathematics Outreach Poster Session, Joint Mathematics Meetings, Boston, MA, January 5, 2012.
    2.3.1. Poster
    2.3.2. Handout
2.4. "Mathematics and Equity, Past and Present, through the Lives and Work of Women Mathematicians," paper by J. Dewar, A. Crans and L. Khadjavi, MAA Contributed Paper Session on Humanistic Mathematics, Joint Mathematics Meetings, New Orleans LA, January 8, 2011.
    2.4.1. PowerPoint Slides
    2.4.2. Handout
2.5. What is Mathematics? A Scholarship of Teaching and Learning Investigation

I. Background Information on the Course

1. History and Rationale for this Course on Women and Mathematics
2. Intended Audience for the Course

I.1 History and Rationale for this Course on Women and Mathematics

Historically, women mathematicians have received little recognition for their contributions to the discipline. As a result, there have been few role models to encourage young women to continue studying mathematics or to pursue math-related careers. Dr. Jacqueline Dewar, Professor of Mathematics, Loyola Marymount University, recalls being in graduate school before she first heard the name of a woman mathematician. In 1978, the publication of Teri Perl's book, *Math Equals*, inspired her to develop a course for liberal arts students about women mathematicians and their contributions to the field. Approximately twenty-five years later, she revised the course to focus more directly on the needs of future K-12 teachers. The goal is that they, in turn, will inspire their students at an earlier age by sharing the knowledge they gain about these powerful role models, their exciting work, and gender-equity concerns related to women's participation in mathematics.

I.2 Intended Audience for the Course

The course, offered as an upper division math elective, is quite interdisciplinary, pulling from mathematics, psychology, women's studies and history. It is often cross-listed with honors, and previously, but not in its current version, with women's studies. It enrolls both students who do and who do not plan to be teachers. In recognition of this broad range of backgrounds and interests, slightly different learning outcomes were developed for 3 distinct audiences: those enrolled under Honors, and those under Math who did or who did not self-identify as future K-12 teachers. Corresponding assessments were crafted for each version. These are described in Section II: Course Design Elements.

II. Course Design Elements

1. Course Goals and Learning Outcomes as a Math Course
2. Course Content
3. Required Coursework, General Grading Scheme, and Texts
4. Adjustments for Enrolling as a Cross-Listed Honors Course

II.1 Course Goals and Learning Outcomes as a Math Course

When taken as a mathematics course, the overarching goals and learning outcomes are as follows:

Overarching Course Goals (fall into four categories):

W: Women Mathematicians in History
To examine the lives and contributions of women mathematicians from the 4^{th} to the 20^{th} centuries

G: Current Gender Issues in Mathematics Education, Achievement & Participation
To investigate current gender issues related to women's skills and participation in mathematics from elementary school through graduate school and their participation in math-related careers

M: Mathematical Reasoning and Communication
To provide students an opportunity to experience "doing mathematics" in a supportive and cooperative environment and to encourage students to be more aware of their own mathematical thinking

R: Research Component
To undertake a scholarly investigation that explores one or more of the above areas

Learning Outcomes (matched to goals, by letter):

Students will be able to

W1. Describe the life and work of 9 women mathematicians and at least one contemporary woman mathematician
W2. Synthesize from these women's biographies common experiences/obstacles faced by women who wished to participate in mathematics and identify factors that enabled their success
G1. Discuss the current situation in the United States regarding women's participation and achievement in mathematics in K-12, higher education, and industry
G2. Read critically articles in journals and newspapers dealing with gender issues in mathematics or science education
M1. Make and investigate mathematical conjectures, develop arguments in support of or counterexamples for those conjectures
M2. Communicate their mathematical thinking clearly to others (peers and teachers) using mathematical language
M3. Analyze and evaluate the mathematical thinking of others
M4. Recognize and appreciate: mathematics as a study of patterns; the critical and distinct roles of inductive and deductive reasoning in developing (new) mathematics; the utility of multiple representations for a single mathematical concept

R1. Demonstrate the ability to conduct an interdisciplinary research project

For Future K-12 Teachers,
an additional specific goal and outcome:

Additional Goal

P: Pre-professional Development for a Future Teaching Career
To develop expertise in addressing equity issues related to mathematics education at the level you plan to each

Additional Learning Outcome for Future Teachers

Future Teachers will be able to

P1. Prepare and share mathematics teaching resources and lesson plans (including appropriate assessments) that reflect equity principles

II.2 Course Content

1) An examination of the biographies of 9 women mathematicians starting with Hypatia, a 4th century Greek born in Alexandria, Egypt, and ending with Emmy Noether, a 20th century mathematician;
2) Mathematical activities related to the work of these women; topics include inductive and deductive reasoning, prime numbers, conic sections, special curves, functions, sequences, series, difference tables, polyhedra, graph theory, and group theory;
3) An opportunity to experience "doing mathematics" in a supportive and cooperative environment. Mathematical investigations will have a variety of entry points;
4) A study of more recent gender differences in mathematics education, achievement and participation. We will examine questions such as: Why are there so few women mathematicians still today, and even fewer women of color in mathematics? Which of the common themes that emerged from examining the lives of women mathematicians past are still relevant today? Are males better at math than females? Do males like math more? What obstacles do women face in the pursuit of a mathematical education or career? What are the experiences of minority women in mathematics?

II.3 Required Coursework, General Grading Scheme, and Texts

COURSEWORK/GRADING: The course grade will be based on the following components:

30% Homework readings, worksheets, postings on course discussion board & paper on gender issues in math ed/participation
30% Research project on a mathematical topic that connects to one of the women in the course or on a gender-related issue in mathematics
20% Quizzes (every 2 or 3 weeks)
20% Poster presentation and final reflective writing assignment

Option 1 (Recommended for future K-12 teachers) K-12 Lesson plan & presentation that incorporates mathematical women or minority role models
Option 2 Biographical poster presentation, using an online electronic poster tool, about a contemporary woman mathematician, including an analysis of similarities and differences of her educational and career experiences from those of the historical women

The lowest quiz will be dropped; quizzes missed with advance notification for a specific commitment or for a documented emergency can be made up near the end of the semester at a day/time to be announced. The following overall averages guarantee you *at least* the respective grades: 90% A-, 80% B-, 70% C-, 60% D.

REQUIRED TEXTS: *Math Equals* by Teri Perl, Addison Wesley, 1978; *Women in Mathematics: The Addition of A Difference* by Claudia Henrion, Indiana Univesity Press, 1997.

II.4 Adjustments for Enrolling as a Cross-Listed Honors Course

When taken as an Honors Course, the Research Component Goal and corresponding learning outcome R1 are replaced by the following very similar goal and outcome that employ the language of Honors program courses.

H: Honors Program - Interdisciplinary Exploration
To undertake a scholarly investigation of an interdisciplinary nature that addresses a question or concern related to the course

Students will be able to

H1: Demonstrate the ability to make connections between disparate disciplines

Depending on the mathematical background of the students, an alternative version of the quizzes may be offered. In the alternative version, there may be fewer or different questions addressing the mathematical content and more questions concerning the historical or gender-related aspects of the course. This befits the interdisciplinary nature of the course and allows us to accommodate fairly the situation when students enroll with widely varying mathematical backgrounds.

III. Course Materials

1. Timeline for Mathematical Topics, Biographies of Women, and Gender Equity Discussions
2. Addressing the Unifying Mathematical Themes
3. Selection of Mathematical Lessons with Worksheets, Lecture Notes, or other Instructional Materials
    3.1. Lesson: Number-Magic
    3.2. Lesson: Functions
    3.3. Lesson: Modular Arithmetic
4. Reading Assignments and Discussion Questions
5. Study Guides for the Quizzes
6. Reports, Projects, & Final Reflective Writing Assignments
    6.1. First Report Assignment: The Women-and-Mathematics Situation
    6.2. Research Project
    6.3. A Contemporary Woman Mathematician Poster & Presentation Assignment
    6.4. Lesson Plan Assignment for Future Teachers
    6.5. Final Reflective Writing Assignment

III.1 Timeline for Mathematical Topics, Biographies of Women, and Gender Equity Discussions

The content of this course includes mathematical concepts, biographical and historical information, and gender equity issues. Because of the course's multi- and inter-disciplinary nature, this course can be challenging to plan.

The course was inspired by Teri Perl's book (1978), which discusses the biographies of nine women mathematicians from the 4th through the 20th centuries, and, for each woman, presents one or more mathematical topics that relate to the woman's contributions to mathematics. For example, the first woman, Hypatia (4th century AD) wrote a commentary on Diophantus' *Arithmetica* and also a treatise*, On the Conics of Apollonius*. Perl (1978) discusses conics, primarily from a geometric point of view. She defines them as intersections of planes and cones and shows how to draw them by using concentric circles to interpret the distance properties of each conic relative to its foci and/or directrix. This provides two very different representations for the conic sections, and dovetails nicely with one of the course's unifying mathematical themes, namely the existence and utility of multiple representations for a single mathematical concept. Perl also devotes one page (p. 27) to a simple example of a Diophantine equation. Perl aimed to present "unfamiliar" mathematics (p. iii) at a pre-collegiate level (p. ii). On the other hand the audience for a collegiate level course can be quite a bit more sophisticated, mathematically speaking. This not only allows but, in fact, challenges the instructor to augment the mathematical discussions found in Perl's book, while still aiming to present "unfamiliar" mathematics. The mathematics that we have connected to and presented with Hypatia extends Perl's material on conic sections and Diophantine equations and adds new material on prime numbers. For example, we discuss the Dandelin sphere proof (http://en.wikipedia.org/wiki/Dandelin_spheres) of the equivalence of the cutting-the-cone definition and the distance-focus definition. Most math majors and future secondary teachers have never seen, or even questioned, whether or how these two very different definitions produce the same curves. The proof requires no more than a knowledge of high school geometry, and hence is accessible to a wide range of students. We also present the Sieve of Eratosthenes and have in some semesters included a prose version of Archimedes' cattle problem (found on p. 184 of "Solving the Pell equation" by H.W Lenstra, Jr., *Notices of the American Mathematical Society* **29** (2): 182-192) for students to ferret out for themselves some of the Diophantine equations (see also http://www.maa.org/devlin/devlin_02_04.html).

Table III.1 contains the list of women mathematicians found in Perl (1978) and the mathematical topics that have been presented at one time or another in the course in connection with these women. Several sample lessons with worksheets are found in Section III.3. The study guides for the quizzes in Section III.5 provide additional insight into the mathematical content of the course.

Woman	Possible Associated Mathematical Topics
Hypatia 355?-415	Conics, Dandelin sphere proof, Diophantine equations, Prime numbers, Triangular numbers, Sieve of Eratosthenes
Emilie du Chatelet 1706-1749	Functions: properties, representations & interpretations Limits, rates of change and derivatives
Maria Agnesi 1718-1799	Symmetry in graphs Witch of Agnesi
Sophie Germain 1776-1831	Soap bubbles, Minimal surfaces, Modular arithmetic: Is division or cancellation possible?
Mary Somerville 1780-1872	Pendulum properties, shape of the earth Cycloid and related curves
Ada Lovelace 1815-1853	Difference tables Numbers in other bases
Sonya Kovalevsky 1850-1891	Sequences & series Zeno's paradox, Fibonacci sequence, geometric series
Grace Chisolm Young 1868-1944	Polyhedra & Euler's formula Mind-reading cards (base 2)
Emmy Noether 1882-1935	Binary operations, groups, symmetry groups Cayley tables and group properties

Table III.1 Nine women mathematicians and associated mathematical topics

Normally the historical/biographical material (and the corresponding mathematics) is presented in chronological order, beginning with Hypatia and ending with Emmy Noether. Students are assigned biographical reading for each woman and required to post comments on an electronic bulletin board prior to in-class discussion of each woman's life and experience. The readings and discussion prompts are found in Section III.4. The goal is to distill common threads or themes from the experiences of these women mathematicians and to consider which of those themes or variants thereof still affect contemporary women mathematicians.

Table III.2 contains a timeline for the Fall 2010 course to show how the various components were scheduled. The two instructors who team-taught the course in Fall 2010 deviated twice from the normal chronological order to accommodate their travel commitments and personal preferences for certain topics.

Week	No. classes	Dates	Topic
1	1	31-Aug	Syllabus, survey on mathematics, intro to course, themes and gender issues, number-magic problems
	1	2-Sep	Triangular numbers, class discussion on reading
2	1	7-Sep	Hypatia bio, conics, applications of conics
	1	9-Sep	Dandelin Spheres
3	1	14-Sep	Sieve of Eratosthenes and primes, finish up Hypatia
	1	16-Sep	Emilie du Chatelet, Enlightenment, functions
4	1	21-Sep	Quiz #1, functions
	1	23-Sep	More on functions
5	1	28-Sep	Sophie Germain, Pythagorean Thm discussion
	1	30-Sep	modular arithmetic, incl. mult, division, & cancel. laws
6	1	5-Oct	Quiz #2, finish Germain
	1	7-Oct	Maria Agnesi, witch problems
7	1	12-Oct	Finish Agnesi
	1	14-Oct	More on Pythagorean Thm (true for other shapes?)
8	1	19-Oct	Mary Somerville, pendulum, shape of earth, cycloid
	1	21-Oct	Quiz #3, revisit induction/deduction, Galileo
9	1	26-Oct	Ada Lovelace, difference tables
	1	28-Oct	Finish Lovelace
10	1	2-Nov	Grace Chisolm Young, polyhedra
	1	4-Nov	Quiz #4, Euler's formula
11	1	9-Nov	Sonya Kovalevsky, sequences & series
	1	11-Nov	Zeno's paradox, geometric series
12	1	16-Nov	Emmy Noether, discussion on marriage, career roles
	1	18-Nov	Quiz #5, race and gender
13	0	23-Nov	no-class
14	1	30-Nov	Noether, groups, symmetry groups
	1	2-Dec	Cayley tables and group properties
15	1	7-Dec	Quiz #6, AWM reading and discussion
	1	9-Dec	Finish AWM, final remarks, closing
16	1	14-Dec	Presentations during Final Exam period

Table III.2 Timeline for Fall 2010

What follows is a short summary of each class in Fall 2010 provided by one of the students in the class.

Women and Mathematics – Class Summaries – Fall 2010

August 31, 2010

Tuesday August 31, 2010 was the first day of class. We started off the class introducing ourselves saying both our name and major, including the professors. Then the syllabus, an outline of the day's lecture, a worksheet on math/gender issues, and an article was passed out. We were asked to fill out the "To Get You Thinking about Math/Gender Issues" worksheet to the best of our knowledge.

Then we went over the syllabus, going over the office hours, coursework and grading and skimming over the different projects we have in store for us this semester. The lecture, titled “The Feminine Side of Mathematics” or “Mathematics, Money, and Sex” was then started. Throughout the lecture we went over the different facts on the blue sheet and some statistical facts from the 1970s on the earning differential, participation in math-related careers, and how boys had more opportunities to study math than girls.

Then we went over two gender differences between males and females. The first was women avoiding math, and the second was math anxiety. For math avoidance we talked about Sells’ 1973 study of incoming freshman into UC Berkeley where 57% of boys had enough math to study science and engineering compared to only 8% of girls. For math anxiety, we went over how it can be a problem for both genders but more women are willing to admit it, among other facts.

Finally we ended the class with talking about why women avoid math. Three possible reasons were attitude, achievement, and aptitude, all of which we found to be false reasons. We then went over stereo-type threat and discussed how a negative stereotype can affect students before taking a test because they gain anxiety about fulfilling the negative stereotype. Finally we watched the video “ETS Field Study about AP Exams” from the MSRI at Berkeley, which showed that asking gender and race before the exam hindered girls performance on the test.

At the end of class we were given a number-magic problem and asked to find an explanation of the pattern we observed for homework.

September 2, 2010

Thursday September 2 we started off the class finishing the lecture from the previous class. We talked about the social/cultural factors that contribute to women’s participation in mathematics. Specifically we talked about the different experiences in the home, for example, who helps them with what type of homework.

Next we went into talking about the number-magic problem we were given at the end of last class and for homework for this class. We showed both the algebraic and an alternate symbolic proof for the problem. Then we talked about the difference between inductive and deductive reasoning and how we use each regarding mathematics. Finally we talked about mathematical notation and wrote a formal proof for the problem. We had a bit of history and found the Descartes and Viete started mathematical notation which implies it has only been around for about 400 hundred years.

We closed out the class by splitting into three groups of four to discuss the articles we had to read for homework, the Halpern and Gordon articles (Halpern, D., Benbow, C., Geary, D., Gur, R., Hyde, J. & Gernsbacher, M. (2007, Fall). “Why do men dominate the fields of science, engineering and mathematics?” *Scientific American.* See http://www.sciam.com/article.cfm?id=sex-math-and-scientific-achievement and Gordon, C. & Keyfitz, B. (2004, August). Women in academia: Are we asking the right questions” *Notices Amer. Math. Soc. 51*(7), 784-786). We discussed the homework questions asked about the Halpern article and are going to continue the questions next class.

September 7, 2010

Today’s class on Tuesday September 7, 2010 started off by finishing the discussion on the articles read for homework, then ended in discussing a little bit about Hypatia. First we got back into our groups from last time and answered the questions: what was the major thesis, how was it supported, and was there anything that surprised you. These questions were discussed for both the Halpern and Gordon article. For the Halpern article we touched on how it is very complicated why there are fewer women in math than men. We also thought it was surprising that Summers who was so educated could say something so “ignorant”. For the Gordon and Keyfitz article we discussed how there are few women in both math degrees and academia. We then went into a long discussion on the difference between R1 and Group 1 schools and the different policies like family friendly and spouse friendly. We also highlighted the statistics that out of the 24% of women who get the PhD’s from a Group 1 school, only 11% of junior tenure track faculty at Group 1 schools are women. Then both Professor Dewar and Professor Crans talked a little bit about their own tenure experience and stories that they have heard from colleagues.

Next we went into talking about the reading we had on Hypatia. We went over how easy it was to just take anything we were reading as truth about the lives of women mathematicians from history. Professor Dewar used to teach facts about Hypatia that newer scholarship has revealed to be in error. We also dabbled into talking about how little we really know about her and her mathematics.

September 9, 2010

On Thursday September 9, 2010 we began class by looking at some of our homework information and questions about conic sections. We then discussed both the distance definition of a circle and of an ellipse and looked at each of the equations x^2+y^2=r^2 and (x^2/a^2) + (y^2/b^2)=1, respectively. Then we talked about how to relate the distance definition and equation to the actual physical conic sections. We then did the worksheet on drawing ellipses and hyperboles that was present in our reading from the night before. Finally we discussed the Dandelin proof on conic section which does end up relating the shape to the formula for the ellipse. We also concluded with discussing that Hypatia would have been interested in conic sections for their beauty and in relation to the “impossible construction” problems of concern to the ancient Greeks. By contrast, in modern times, knowing about conic sections is essential when we study astronomy and GPS technology.

September 14, 2010

On Tuesday September 14, 2010 we started class by looking at the difference between xyz and the 123. The first usually means x*y*z and the second means 1(100)+2(10)+3(1). We then went through the algebraic proof to get the distance equation of sums for an ellipse and had a small discussion on the applications of the ellipse. We then looked at the figurate numbers including the square, triangular, and rectangular numbers. We provided a geometric, algebraic, and numerical representation for all three. Finally we looked at representing the nth triangular number t(n) = 1 + 2 + 3 + … + n, both in sigma notation and recursively, and discussed how we would be able to add a number of numbers together in a fast way. The story of how Gauss added the numbers from 1 to 100 was told and then we showed how to get the formula t(n) = n(n+1)/2 on our own. Then, we finished the class with a discussion on primes, mainly what they were and how they can be represented as a rectangular number.

September 16, 2010

On Thursday September 16, 2010 we started off class with some discussion of topics from last time and then moved into a discussion on prime numbers. We looked over the Sieve worksheet we completed for homework and talked about the series s(n) being the nth square numbers and t(n) being the nth triangular number. Next we looked at the question, “Why is 1 not prime?” To answer we stated the Fundamental Theorem of Arithmetic: Every integer n>=2 is either prime or can be factored uniquely as a product of primes up to reordering factors. Then if 1 were prime it would defy the unique part of the FTA. Next we looked at Mersenne and Germain primes. Finally we ended with looking at our homework answers to the professors' question on how to add all the numbers up to ten in a creative way and with looking at the proof for the sieve conclusion: if m is composite and m<n^2, then one of m's factors must be less than n.

September 21, 2010

Tuesday, September 21, 2010 we began class with a quiz then went into sharing our thoughts on the newest woman we read about for homework Emilie du Chatelet. We commented on her lifestyle and similarities she had with Hypatia. Next Professor Crans asked each of us to share our favorite function and wrote them all out on the board. We then had a discussion on what makes a function a function and came upon the definition: a function is a rule that assigns to each input exactly one output. We also talked about the vertical line test and gave the example y^2=x as a non-function. Next we did the worksheet on functions where we were told a situation and had to assign a function from it. We voted anonymously for our choice by texting our choice using the website http://polleverywhere.com. We then discussed and argued for our choices with the rest of the class and came upon the conclusion that one graph can describe many different things and that one story can have many different graphs depending on how one interprets the information.

September 24, 2010

On Thursday, September 24, 2010 we started class by going over the quiz we took last class. We went over the ellipse question and discussed again how to determine whether a point is actually on the ellipse. For the specific problem we looked at the Pythagorean theorem to find the distance sum for the point in question. Then we got into partners and discussed the page of the function worksheet we needed to do for homework which involved sports and whether they fit onto the given graph. We had a discussion in class and again found that many different situations can fit one graph and vice versa. Next Professor Crans had us look at a table for which she would put an input then an output and we had to guess the function. The first function was f(name)=first consonant in the name, where we then had to discuss what we would do if the name did not have a consonant. This is an unusual situation to encounter the idea of a domain of a function. She did another example with numbers and we discussed the fact that we used inductive reasoning to figure out the function. Finally we had a function summary where we found that a function can be expressed algebraically, graphically, tabularly, and in words.

September 28, 2010

On Tuesday September 28th, we started off the class talking about the commonalities between Sophie Germain and the other women we have learned about thus far. For example Sophie studied philosophy and psychology, linguistics, and had male mentors in her life. Next we delved into Germain's mathematical work in number theory. Goldbach's conjecture was explained before we started into Fermat's Last Thereom, which states: For integers n>=3, there do not exist nonzero positive integers a, b, and c such that a^n + b^n = c^n. We then discussed how the Pythagorean thereom is closely related because it is when n=2. Then we learned the definition of divisibility and went over examples and non-examples of when one number divides another. Finally we ended the class with some questions about divisibility concluding that if a does not divide b then b is not a multiple of a, and with starting to learn about modular arithmetic by looking at the clock. All of this will then lead to how Sophie proved an important part of Fermat's last Theorem dealing with specific prime numbers.

September 30, 2010

Thursday September 30th, we began class by reviewing the idea of modular arithmetic and specifically looked at numbers congruent to each other modular 3. We began looking at the numbers 2, 5, 8, and etc. and found that the division algorithm shows how all of them have a remainder of 2 upon division by three and are therefore all congruent modular 3. This then led us into the notion that with modular arithmetic we can partition the integers. For example, back to modular three, we can partition the integers into three sections, those congruent to 1 modular 3, those congruent to 2 modular 3 and those congruent to 0 modular 3. Next we were split into partners and asked to do some examples with a partner computing different values in modular arithmetic. We then found through those examples that a congruent to b modular n is the same thing as saying n divides (a-b), or that (a-b) is a multiple of n. This then connected our lesson before about divisibility to this new notion of modular arithmetic which will then lead us to understanding Sophie's contribution to Fermat's Last Theorem a little better.

October 5, 2010

Tuesday October 5, the class was started by a quiz and then by looking at the homework we did to write an elementary school word problem involving division. We came to the conclusion that no "partitive" word problem can be written for division by a fraction. Next we finally started talking about Fermat's Last Theorem and how one would go about solving the problem. First we decided that we could split it up into cases and go from there. Our first case was: consider n to be a prime number that does not divide a, b, or c in regards to the previous equation (a^n + b^n = c^n). Then we considered a second case which was that n is a prime number that divides at least one of a, b, or c. After thinking about these cases we came to the Sophie Germain Theorem: let p be an odd prime. If there exists a prime q such that 1) If a^p + b^p is congruent to 0 (mod q), then q divides a or q divides b or q divides c; 2) there does not exists an integer x such that x^p is congruent to p (mod q). With that hypothesis then our case 1 of Fermat's last theorem is true for n=p. Sophie first proved that case one held for prime p with the property that 2p+1 is also a prime. This specific type of prime number is named a Germain prime.

October 7, 2010

Thursday October 7th, we began class by looking at Fermat's Little Theorem. This states that if n^(p-1) is congruent to 1 modular p, then n^p is congruent to n modular p. This led us to look at the condition if ax is congruent to ay modular n. We found that is does not follow that x=y but perhaps it does follow that x is congruent to y modular n? This question was left for us to think about for our homework assignment. Next we discussed our discussion board assignment and the next woman that we will be learning about, Maria Agnesi. We discussed her life and went over similarities to the other women such as the fact that her father was a mathematician like Hypatia and she was wonderful at languages.

October 12, 2010

Tuesday, October 12th's class focused mainly on discussing the actual mathematical work of Maria Agnesi. First we took a look at the witch of Agnesi properties from both a mechanical and an equation approach. Then we delved into this concept of producing new math with inductive and deductive reasoning and the "what if not" tools. First, in beginning to study the witch of Agnesi, we agreed that there is an intrinsic difference between drawing a circle with a compass and drawing a line with a ruler, where tracing a can would be equivalent to drawing a line with a ruler. Then we looked at online graphs of the curve the witch of Agnesi and we decided that we have 2 equation possibilities based on asymptotes, the shape, symmetry, and the fact that points on the graph only contain positive values for y. The two possibilities we were deciding between were an exponential function or a rational function with even powered polynomials. We then finally did the actual derivation and found that we were right, it involved a rational function with an even powered polynomial in the denominator.

October 14, 2010

Thursday October 14th, we began class a little differently by writing down what math is mostly about in one word then explaining why. Next we went over our witch of Agnesi homework. To find the max we would use x=0 because we need the denominator to be as small as possible. For the asymptote, we would want to observe when the absolute value of x is extremely large and find that the equation tends toward zero. Next we discussed our modular arithmetic homework from a while back, and found that if ax is congruent to ay modular n, x does not have to be congruent to y. We found that additional hypotheses are necessary to conclude anything else about the statement. Next we went into generating new math from what-if-not thinking. Using the Pythagorean Theorem, we looked at things we could change like the area of the squares, shape of the squares, shape of the right triangle, or even the operation being performed. From this type of thinking we then came up with a conjecture, in any isosceles triangle the sum of the areas of the squares on the legs equals the area of the square on the base, and proved the conjecture false using previous knowledge from geometry. We then tried the conjecture that for any right triangle the area of the semi-circle on the hypotenuse equals the sum of the areas of the semicircles on the two legs and showed that was true, again using previous knowledge. In this way, we were able to use What-if-not thinking and deductive reasoning to generate new math.

October 19, 2010

On Tuesday October 19th we started off by talking about how to describe continuity without calculus. We discussed three rules for continuity, one that f(x*) exists, two if the limit as x approaches x* of f(x) exists, and three that the limit is equal to f(x*). Next, we re-visited our question if ax is congruent to ay(mod 12) then is x congruent to y(mod12), and found that a and 12 need to have no common factors. In other words the cancellation rule does not hold for this equation and therefore with modular equations. Finally we ended class with discussing the life of Mary Somerville, the next woman mathematician we learned about. We talked about how she married one of her second cousins for her second husband and it was only then that she had access to any type of book. We also found it interesting that she managed to not go against any of the social structures of her time period. She learned both Latin and Italian and therefore had the affinity for languages in common with the other women mathematicians. Finally we said she was a science expositor and compared her to Bill Nye, the science guy.

October 21, 2010

On Thursday October 21st we had a full class talking about prove-disprove logic, the search for a better clock, and the properties of a cycloid. First we talked about needing to work from something true to arrive at a truth. It was the final truth that one had then proven. We saw it was possible to start from something false and end at a truth statement, this proves nothing, certainly it does not prove that the original statement was true. Then we went into talking about the pendulum clock and the search for the curve that would make time isochronous or in other words the pendulum clock more accurate. We then talked about how the shortest distance between two points is a line but the fastest is the cycloid. This then led to the discovery that the pendulum should swing in a cycloidal arc. To achieve this curve, the clock makers discovered they could put cycloidal jaws at the top to make the pendulum swing in a cycloidal curve.

October 26, 2010

Class on Tuesday October 26th started off by discussing our next woman in math, Ada Lovelace. We then went back into our discussion that has been going on throughout the semester about inductive and deductive reasoning. For this we looked at the problem if there are n number of points on a circle then how many regions will result from connecting the points. For this we found results for 5 points and used inductive reasoning to predict the results for 6. But, we found that our prediction was wrong, which shows that inductive reasoning is not always accurate. Then we went into talking about function machines, which relates to the math that Ada Lovelace did during her mathematical career. We showed functions by a table, a graph, and by an algebraic function. This relates to how we have been talking about different mathematical representations for things such as functions in mathematics. Then we did some difference tables and found that if f is linear then the first difference is constant. We also found that for quadratic functions that the second difference was constant and for cubic functions that the third difference would be constant. In conclusion, if a polynomial is of degree n, in its difference table the nth difference will be constant.

October 28, 2010

This Thursday's class we began looking at a table that was similar to those we had done last class that represent a function. We then did a worksheet with a partner on finding difference tables and looking to find what the polynomial could be for the data that we had. After finishing that worksheet and making sure we understood the difference tables, we finished up our discussion on the cycloid and went over the "Incredible Cycloid" hand-out which gave us cool facts on how the arc of a cycloid was measured and how its area was measured.

November 2, 2010

This Tuesday November 2nd we started class by returning to our number of regions in a circle problem and decided that real mathematicians would look for a formal proof to determine the true answer to the problem. Then we started talking about our next famous mathematician, Grace Chisholm Young and actually saw a picture of Girton College, which she attended. Grace was the first mathematician we read about to receive a university education. She had to balance raising six children and her career and many question how much of her husband's work was actually her own. A fun fact we learned was that she has a granddaughter that is still alive and is also a mathematician today!

Next we talked about the math that Grace Chisholm Young liked to do and was famous for. We discussed the 5 platonic solids made up of regular polyhedral. We said that regular polygons have equal sides and angles. Some of the solids include the icosahedrons, the tetrahedron, and the octahedron. The platonic solids were named after Plato who associated the elements of the universe with the shapes. For example cube is Earth, tetrahedron is fire, and octahedron is air.

November 4, 2010

Thursday, November 4th we began class where we left off last class looking at different polyhedral which relates to Young's work with the platonic solids. We each got a 3D figure in class and were asked to count the number of vertices, edges, and faces and record it on the board. After everyone finished their counting we looked at all the data and tried to find a pattern. We found Euler's Formula which is $V+F-E = 2$, where V stands for number of vertices, E for number of edges, and F number of faces. This implies that 2 is the Euler characteristic of polyhedron. The Euler characteristic is an invariant of polyhedral. Other invariants include cardinality of sets and dimensions of vector spaces. Euler's formula only holds for simple polyhedral, in other words polyhedral with no holes.

November 9, 2010

Tuesday November 9th the class began with elevator speeches, which are 1 minute talks about our research papers and the progress we are making with them. We then did a small recap of the Euler Characteristic and then moved on to talk about our next famous women mathematician Sonya Kovalevsky. We found that she, along with all of our other famous mathematicians, had a male mentor encourage her in mathematics, her Uncle Peter and the famous Weierstrass. Another trait she had in common with the other famous mathematicians was that she was also good at languages. Sonya became a tenured professor at a University in Sweden and her major accomplishment is the Cauchy-Kovalevsky Theorem. Next, we looked at some of the mathematics Sonya liked, which are sequences. We talked about Zeno's paradox and the notion of an infinite series. We defined a sequence to be an ordered list of elements, while a series is the sum of a sequence. For a numerical approach we looked at a table with the # of the term in one column and the partial sum in the other. Then we looked an algebraic approach which gave us the same answer for the sum of the sequence.

November 11, 2010

Thursday, November 11th, we continued from last class and looked at a geometric representation of Zeno's Paradox and still found the same answer for the sum of the series $1 + ½ + ¼ + \dots$. We found that we will never need to shade more than the square and that we would eventually shade in the whole square. We then gave ourselves three goals, 1) to find the definition for these kind of sequences and series, 2) find a formula for the nth partial sum, and 3) to find a formula for the sum. We then defined the geometric series $S = a + ar + ar^2 + \dots$ and looked at how .99999 repeating is a geometric series whose sum is 1. We then found that $S= a/ (1-r)$, and looked at both divergent and oscillating series. We then found that our sum formula only works if the absolute value of r is less than 1.

November 16, 2010

On Tuesday, November 16 h, we continued to work on a geometric series used to model a problem involving mercury poisoning that occurred in Iraq in 1972 due to the consumption of homemade bread made from contaminated wheat. (See Problem #65 on pages 499-500 of Zill and Wright's Single Variable Calculus with Early Transcendentals, 4th ed., Jones and Bartlett, 2011.)

November 18, 2010

On Thursday, November 18 h, we had a guest lecturer, Dr. Lily Khadjavi, on the topic of Race and gender: A double whammy. We were to have read Henrion's text *Women in Mathematics,* pages 188-233. prior to class.

November 23, 2010

Class on Tuesday November 23rd started with a discussion on our next famous woman, Emmy Noether. We pointed out that she made ground breaking contributions to the field of abstract algebra, more specifically ring theory. Then we learned a little about the history behind group theory which is a type of abstract algebra. We found out that LaGrange and Abel worked with groups before they were formally defined and Galois coined the actual word “group” in the early 1800s. Group theory was firmly established in the 1870s and finally in 1983 the finite simple groups were completely classified. Next we looked at some actual group theory by looking at a square and its rotations. We also found by inductive reasoning that the number of sides the shape has is that same as the number of rotations it has. For example a square has four sides and four rotations. The square also has flips and can be flips horizontally, vertically and diagonally in two ways. These rotations and flips together are called the symmetries of the square and form a group. Then we were each given our own shape and asked to find the rotations and flips and their combinations. We discussed with partners and found some similarities with some of the shapes.

November 30, 2010

On Tuesday November 30th we picked up where we left off and observed that the rotations and flips we looked at are called “rigid” motions and symmetries. We also noted that the notation of these symmetries is extremely important for the tables we make with their combinations which are known as Cayley Tables. We then wrote out the tables for addition modular 4 and addition modular 6. Finally we came to the definition of a group which is: Let G be a set. Let * be a binary operation on G. We say G together with * is a group if the following properties hold: 1) closure, if a and b are elements of G then so is a*b, 2) identity, there is an element e such that a*e=a, 3) inverses, for every element a, there is an element b such that a*b=e, 4) associativity, for every elements a,b,c in G a*(b*c) = (a*b)*c. Some group examples we could think of were integers with addition, and even integers with addition.

December 2, 2010

Thursday, the 2nd of December, we first recalled what the definition of a group was and that the satisfying conditions were closure, identity, inverse and associativity. Then we looked at some non-examples like integers with multiplication which fails due to the inverse condition. Then we looked at the counting numbers with addition which fails both the identity and inverse conditions. We also looked at odd integers with addition which fails closure and identity. Finally we looked at integers with subtraction which fails the associativity condition. We then went over our homework which involved the mattress group and found that every element was its own inverse. Then we learned that commutative groups are those in which a*b = b*a and those groups are also called Abelian. So the mattress group from our homework was Abelian, while the symmetries of the triangle are not. We also closed up our discussion on Emmy Noether by noticing she too had a father that was a math professor and had male mentors such as Hilbert and Weil.

December 7, 2010

This Tuesday, December 7th, is our last day of class before our paper presentations and the end of the semester. We started off by looking at two different Cayley tables and trying to determine whether they were a group or not. One table did not have inverses while the other was a group. We also found that we still had the unanswered question of what is the formula for the number of regions in a circle when connecting n points and looked at a worksheet to answer that question. Finally we talked about our reading on the AWM or the Association for Women in Mathematics. We discussed how Mary Gray was very well prepared to effect change which was badly needed. We also found that the AWM created a women-in-math community, which provides essential support for women in mathematics.

III.2 Addressing the Unifying Mathematical Themes

As described in Section III.3, for each of the nine women mathematicians, mathematical ideas related to their work are presented in the course. (See Table III.1.) The result is a potpourri of topics that meanders across conic sections, functions, special curves, modular arithmetic, difference equations, group theory and more. To unite these disparate topics, we find and emphasize three common themes:

1. Math is, at its core, a study of patterns;
2. Inductive and deductive reasoning have critical and distinct roles in developing (new) mathematics;
3. A single mathematical concept can have multiple representations that serve to illuminate or justify its properties

The first of these, that mathematics is best described as a study of patterns, not numbers, flows naturally from the second. The success of the course in shifting students away from the belief that math is a study of numbers has been examined. (See Dewar (2008), available at
http://sigmaa.maa.org/rume/crume2008/Proceedings/Proceedings.html.)

As indicated in Table III.2, the first mathematical topic presented in the course is "number-magic problems." As an example, students are told to:

- •Take a number
- •Add 3
- •Double the result
- •Add 4
- •Divide by 2
- •Subtract your original number
- •The result is -----.

This elementary problem provides fertile ground for introducing the three themes on the first day of the course. A pattern is immediately encountered when every student, regardless of what initial number they choose, arrives at the number five. Through inductive reasoning the class forms a conjecture: *Regardless of initial starting number, the sequence of operations will always produce the number five*. The conjecture is then verified by means of deductive reasoning, which we can represent in two ways, using the normal algebraic notation or by means of a picture proof as follows.

<u>Algebraic Proof</u>	<u>Picture Proof</u>
Let x be any starting number.	Let ■ represent any number and • represent 1.

Then each successive operation results in the following:

x + 3	■ • • •

2(x + 3) or 2x + 6	■ • • • ■ • • •
2x + 10	■ • • • ■ • • • • • • • or ■ • • • • • ■ • • • • •
x + 5	■ • • • • •
5	• • • • •

More challenging puzzles can follow to reinforce the themes. (See Section III.3 for a further discussion of this lesson.) For most students, the picture proof is already "unfamiliar" mathematics. Furthermore, the division step in the picture proof invites a discussion of the two different interpretations of division (measurement and partitive), typically encountered in mathematics for elementary teachers courses but rarely, if ever, in mathematics courses for majors.

The following examples illustrate opportunities for the unifying themes to emerge from the various mathematical topics in the course.

Examples of Paired Occurrences of Inductive and Deductive Reasoning

- Formulating conjectures about **number-magic puzzles**, based on patterns seen in examples, and then proven deductively.
- Observing a pattern in the **Sieve of Eratosthenes** when answering the question: After removing all multiples of primes up to but not including p, what is the smallest remaining composite? Proving the pattern to hold for all primes p.
- Looking at **triangular numbers** (1 + 2 + … + n) for small values of n, and guessing an algebraic formula, then proving, the formula to be true both algebraically and geometrically.
- Seeing a **limit** approached numerically when computing a difference quotient for smaller and smaller denominators. For many simple polynomial functions, the difference quotient can be written as an algebraic expression and simplified to "justify" the limiting value that was obtained numerically.
- Examining cases where cancellation does and does not hold in **modular arithmetic** (when does ac = bc mod n implies a = b mod n?), then (depending on the background of the students) proving the resulting conjecture.
- Observing that the nth row in a **difference table** for an nth degree polynomial function is constant and proving that this is true for small (or all) values of n.
- Observing properties of the **Fibonacci sequence**, then (depending on the background of the students) proving the resulting conjectures.
- Observing patterns in **sequences and series**, then proving the resulting conjectures.
- Discovering **Euler's formula** by counting faces, edges and vertices on a number of polyhedra, then examining a deductive proof scheme (See http://plus.maths.org/content/os/issue43/features/kirk/index which recaps material found in Volume 1 (pp. 581-599) of James R. Newman's *The World of Mathematics*, published by Simon and Schuster, 1956.)
- Examining the number of **regions formed within a circle** when all chords are drawn connecting n points on the circumference and discovering that the pattern observed up through n = 5 points (2^{n-1} regions) fails for n = 6 points. The ensuing discussion underscores the point that patterns must be justified, not just observed. This example reinforces the idea that inductive reasoning serves to discover conjectures, not results. Results are proven using deductive or other logical arguments. Then the actual pattern is discovered and proven via a

different analysis (See Shultz, Harris S, Janice W. Schultz, and Richard G. Brown. (2003). Unexpected answers. *Mathematics Teacher, 96*(5), 310-313.)

Examples of Multiple Representations

- **Square/Rectangular/Triangular/Polygonal numbers**: numerical sequence, algebraic formula, dots arranged in corresponding polygonal formation, numerical descriptions (See also Steven Strogatz's blog about groups of rocks http://opinionator.blogs.nytimes.com/2010/02/07/rock-groups/)
- **Justifying formulas for the nth polygonal number:** algebraically and geometrically
- **Models for multiplication of numbers:** Interpreted as repeated addition or as the area of a rectangle
- **Prime numbers**: numerical description using divisibility property, geometrically as dots that can only form a straight bar not a rectangular array, impossibility of representing a prime number as the area of a rectangle with integer sides, unless one side has length one
- **Functions: (**algebraic or other) formula, table/chart of values, graph in the Cartesian plane, the path swept out by the motion of some point (e.g., cycloid curve)
- **Pythagorean Theorem:** algebraic statement referring to squaring the lengths of the sides, geometric statement referring to areas of squares erected on the sides
- **Number magic games**: algebraic proof and picture proof (See Section III.3)
- **Definition of the circle:** Geometrically as the set of all points equidistant from a single fixed point versus the set of all points satisfying a certain equation
- **Definitions of the conic sections:** Cutting the cone definition, Distance from foci definitions
- **Representations of the conic sections:** Equations, graphs, intersections of a plane and a cone
- **Proof of Euler's Theorem for polyhedra:** The theorem which is a statement about properties of 3-dimensional polyhedra is replaced with a corresponding statement about 2-dimensional networks, justified in that domain, and then converted back to the 3-dimensional statement.
- **The degree of a polynomial pattern:** Represented by the highest degree exponent its formula and by the level of its constant difference in a difference table
- **Expressing numbers in a given base:** using positional notation [such as 23 in base 10] or expanded notation [such as (2x10) + (3x1)]
- **Expressing numbers (numerals actually):** in different bases
- **Geometric series:** Find sums numerically, algebraically, geometrically
- **Commutativity of a binary operations** Expressed by the operational notation a*b = b*a and by symmetry in the Cayley table
- **Identity element for a binary operation:** Expressed by the operational notation a*e = e*a = a and by a column and row in the Cayley table matching the boundary row and column
- **Inverse element for a binary operation:** Expressed by the operational notation $a*a^{-1} = a^{-1}*a = e$ and by finding the occurrence of each element once and only once in each row and column of the Cayley table

III.3 Selection of Mathematical Lessons with Worksheets, Lecture Notes, or other Instructional Materials

III.3.1 Lesson: Number-Magic

Instructor's Commentary

This introductory mathematics lesson, on the nature of mathematics, is designed to span two class periods, the first and second periods of the semester, with most of the lesson presented in the second period. Here is the explanation for this design. The first (75 minute) class period of the semester opens with a discussion of the nature of the course; that it is not just a mathematics course, but rather is very interdisciplinary in content, drawing on gender studies related to mathematics and science achievement and participation, as well as history of women in mathematics. The course syllabus and learning outcomes are presented, and students are introduced to the research on gender differences in mathematics dating back to the 1970s. To emphasize the variety of content to be considered in the course, at the very end of the first period a small amount of time is devoted to a simple mathematical activity. Students individually produce some data and collectively observe a pattern, in a little number-magic game. They are asked to consider why the pattern occurs and told to come to the next class prepared to discuss that. In the following class, students will discuss why the pattern holds and will encounter three ways of representing a justification that the pattern always holds, one of which is almost certainly going to be new to them. The homework assignment given at the end of the second class is designed to address each of the three mathematical themes. It provides practice on similar number-magic games and includes a writing prompt that specifically encourages student reflection on the mathematical themes.

<u>Mathematical Themes Addressed</u>

- Math is, at its core, a study of patterns
- Inductive and deductive reasoning have critical and distinct roles in developing (new) mathematics
- A single mathematical concept can have multiple representations that serve to illuminate or justify its properties

<u>Connections to the Work of a Woman Mathematician:</u>

This lesson is not intended to connect to the work of one of the women mathematicians.

Mathematical Learning Outcomes

Students will be able to:

- Make and investigate mathematical conjectures arising from simple number-magic games, and develop arguments in support of those conjectures (M1*)
- Communicate their mathematical thinking clearly to others (peers and teachers) using standard mathematical language and also a novel mathematical notation (M2 and M3*)

- Engage in mathematical discussions with classmates and the instructor that leads them to analyze and evaluate the mathematical thinking of others (M3*)
- Recognize and appreciate: mathematics as a study of patterns; the critical and distinct roles of inductive and deductive reasoning in developing (new) mathematics; the utility of multiple representations for a single mathematical concept (M4*)

* These refer to the course learning outcomes found in Section II.1.

In-class

The instructor reads to the entire class the following instructions, one step at a time.

Each student should take out a piece of paper and:

1. *Write down a number, any number (It can be small or large, and you will do some arithmetic with it.)*
2. *Add 3*
3. *Double the result*
4. *Add 4*
5. *Divide by 2*
6. *Subtract your original number*

The instructor then asks a few students, one at a time, what result they got, and after getting several reports of five, what was the starting number. A pattern is immediately encountered when every student, regardless of what initial number they choose, reports (or should report) the number "five." (NOTE: Instructors should be prepared with a supportive comment in case of hearing an erroneous answer resulting from an error in calculation or misunderstanding a step.) Encourage the class to form a *conjecture* that describes the pattern they observed and write it down for everyone to see: *Regardless of initial starting number, the sequence of operations will always produce the number five*.

The observation of the pattern and the formation of this conjecture should be identified as *inductive reasoning*. Homework assigned at this point is to try to determine why everyone got five.

Upon return to class, students' should discuss their reasons why with at least one other student and then the whole class can produce a typical algebraic proof (see left column in Figure III.1 below). The instructor should label this type of justification *deductive reasoning* and engage the class in a discussion of the different roles for inductive and deductive reasoning in mathematics. At this point the instructor can introduce unfamiliar notation to provide another proof (shown in the right column in Figure III.1). The proof with either notation can also be written in paragraph form, using complete sentences, if providing students practice in writing proofs in paragraph format is desired. The result is three different representations for the proof.

Algebraic Proof	Picture Proof
1. Let x be any starting number.	Let ■ represent any number and • represent 1.

Then each successive operation results in the following:

2.	x + 3	■ • • •
3.	2(x + 3) or 2x + 6	■ • • • ■ • • •
4.	2x + 10	■ • • • ■ • • • • • • • or ■ • • • • • ■ • • • • •
5.	x + 5	■ • • • • •
6.	5	• • • • •

Figure III.1 Algebraic and Picture Proofs for a Number-Magic Problem

This lesson allows for some additional connections to unfamiliar mathematics. For example, unless students have had a course in the history of mathematics, they will be unaware that algebraic notation is only about 400 years old, something they are sure to find surprising. They can be asked to think about how Pythagoras might have written or thought about the Pythagorean theorem when there was no algebraic notation (something discussed later in the course). In addition, the division step (Step 4 in Figure III.1) in the picture proof invites a discussion of the two different interpretations of division, measurement and partitive. The second of the picture proof representations for 2x + 10, namely ■ • • • • • ■ • • • • •, is set up to apply the partitive interpretation of division. These two different interpretations of division are typically encountered in mathematics for elementary teachers courses but rarely, if ever, in mathematics courses for majors. The homework assignment for this lesson follows.

After-class

Figure III.2 provides the homework assignment for this lesson.

Number-Magic (Inductive and Deductive Reasoning) Worksheet

Directions: In Problems #1 and #2 below, follow the directions (going down the page vertically) with <u>at least</u> two different numbers, more if necessary (lines are provided for three different numbers), to find a pattern. Use inductive reasoning to state a conjecture. Then attempt to prove your conjecture is true by deductive reasoning, first, using a squares and dots picture representation; second, using algebraic notation. Next, for either one of the conjectures, write a proof with full sentences in a paragraph format. Finally, describe in your own words, this process of 'doing mathematics' and what you learned about different possibilities for mathematical notation.

Problem #1

1. Choose a number _____ _____ _____

2. Add the number that is one greater than your number _____ _____ _____

3. Add seven ______ ______ ______

4. Divide by 2 ______ ______ ______

5. Subtract your original number ______ ______ ______

6. The result is ______ ______ ______

State a conjecture: ____

Use deductive reasoning to attempt to prove your conjecture true:

Squares and Dots Picture Proof — Algebraic Proof

1.

2.

3.

4.

5.

6.

Problem #2

1. Choose a number ______ ______ ______

2. Triple it ______ ______ ______

3. Add the number that is one greater than your number ______ ______ ______

4. Add eleven ______ ______ ______

5. Divide by four ______ ______ ______

6. Subtract three ______ ______ ______

7. The result is ______ ______ ______

State a conjecture: ____

Use deductive reasoning to attempt to prove your conjecture true:

Squares and Dots Picture Proof — Algebraic Proof

1.

2.

3.

4.

5.

6.

Next, choose one of the conjectures above and write a proof of it with full sentences in a paragraph format.

Finally, describe in your own words, this process of ‘doing mathematics’ and what you learned about different possibilities for mathematical notation.

Figure III.2 Homework Worksheet for Number-Magic Lesson

Postscript

Although this lesson is designed to be introduced at the end of one class period to allow students to seek an explanation for the pattern on their own before the next class period, it could be presented in a single class period. To employ the same pedagogical design, the instructor should allow three to five minutes of time in class after the first pattern is observed for students to try to determine on their own (or in pairs) why everyone got five.

III.3.2 Lesson: Functions Available as a separate pdf file at http://myweb.lmu.edu/jdewar/wam/files/functionlesson.pdf

III.3.3 Lesson: Modular Arithmetic

Instructor's Commentary

This lesson builds on the life of Sophie Germain and number theory, one of her areas of specialty. We begin with a discussion of her life, including the dramatic fact that she posed as "Monsieur LeBlanc," sending homework in to the Ecole Polytechnique at a time when women were not allowed to study there. Her life and access to education can be compared and contrasted with the other women studied thus far in the course. For example, she had some access to education, even if not the same family support, and she had male mentors, notably LaGrange and Gauss. Her accomplishments include work towards "Fermat's Last Theorem" as well as research in acoustics and elasticity, which won a prize from the French Academy of Sciences.

Germain's research in number theory is the motivation for a lesson in modular arithmetic. Once learning how this arithmetic works, students can try many examples, and in particular certain examples are designed to point to a pattern – ultimately leading to conjecture the result known as "Fermat's Little Theorem."

For the more mathematically ambitious, access to computer software such as Mathematica and a bit more work with modular arithmetic opens the door to modern topics, such as the encryption system known as RSA.

<u>Mathematical Themes Addressed</u>

- Math as a study of patterns
- Inductive and deductive reasoning have roles in developing mathematics

<u>Connections to the Work of a Woman Mathematician</u>

As note above, this lesson connects to the work of Sophie Germain.

Mathematical Learning Outcomes

At the conclusion of this lesson, the successful student will be able to:

- Make computations using modular arithmetic and communicate these to others (M2)
- Observe patterns in certain computations and make conjectures about modular arithmetic (M1)
- Engage in mathematical discussions with classmates and the instructor that leads them to analyze and evaluate the mathematical thinking of others (M3)

In-class

Sophie Germain (1776-1831)

- Germain was French, from well-off family, and noted for a singular devotion to mathematics

- Her parents did not support her education (tutoring, etc.) and strongly as upper-class families might have, had they been of the aristocracy rather than middle class. Aristocratic women were expected to have knowledge of subjects such as mathematics for polite conversation.
- Women not allowed in the Ecole Polytechnique (opened 1794, Paris); Germain obtained notes and homework assignments from a friend and posed as student M. LeBlanc to submit her homework
- Germain corresponded with LaGrange (who, happily, was impressed with her work and not upset to discover Germain was a woman) and Gauss (who, apparently, she protected from occupying soldiers; through this, her identity as a woman was revealed to him)
- Her research was in number theory and in acoustics and elasticity (Partial Differential Equations),
- She submitted a paper to French Academy of Sciences that won a prize (formulating a mathematical theory to explain the movement of vibrating plates)
- Germain died before being awarded an honorary doctorate from University of Gottingen

Modular Arithmetic

We begin by introducing students to "clock arithmetic." (When is 11 plus 3 equal to 2? When it is 11 a.m., and you ask someone to meet you in 3 hours, which is 2 p.m.. Thus we are used to working with numbers modulo 12.)

We define equivalence modulo n:
$a = b \bmod n$
if and only if
$(a-b)$ is divisible by n

And note that this is true if and only if
$a = b + kn$ for some integer k

Make many (many) sample computations and use the different formulations above.

Some other terms to define before the end of class: prime (students should already know this one), relatively prime.

The assignment below builds on these ideas to conjecture Fermat's Little Theorem, and we might have students tackle the first two problems in class, leaving the rest for "After Class".

After-class

The following problems may be assigned, or started in class and finished later.

1. Find at least three different numbers that are equivalent to the following:

27 mod 12 = ?
27 mod 5
27 mod 4

2. Compute the powers given, reducing the answer to a small one (depending on the modulus you are using) and then use your answers to make a conjecture. For example, 8 mod 3 can be simplified to 2 mod 3.

2^2 mod 2 = ?
2^3 mod 3
2^4 mod 4
2^5 mod 5
2^6 mod 6
2^7 mod 7
2^8 mod 8
2^9 mod 9

(Note that the modulus changes each time.)

Do you notice a particular result when the modulus is 3, 5, or 7? What do they have in common – what is the pattern?

Use inductive reasoning to make a leap: make a conjecture about the 2^p modulo p, when the modulus is prime.

3. The following message was intercepted. You suspect that the message is in English and that each letter is given by a two-digit number, e.g., A=01, B=02, C=03, D=04, ..., Y=25, Z=26. However, the message below appears to have been encrypted with a shift.

Deduce what shift was used and decrypt the message (if the number x was encrypted as x+k mod 26, where k is the key shift, deduce the most likely k and decrypt).

25 13 10 24 10 08 23 10 25 19 06 18 10 14 24 13 04 21 06 25 14 06

4. Prove that if x is any integer, either x^2 = 0 mod 4 or x^2 = 1 mod 4.
(Suggestion: work out 4 general cases.)

5. Compute the powers given and use your answers to make a conjecture:

a. Compute the powers given below. You may use a calculator or any combination of methods; ideally, you should find some ways to simplify the calculation as you are going through it. (Note that the modulus changes each time, not just the power. Show scratchwork in your write-up.)

$3^2 \bmod 2$
$3^3 \bmod 3$
$3^4 \bmod 4$
$3^5 \bmod 5$
$3^6 \bmod 6$
$3^7 \bmod 7$
$3^8 \bmod 8$
$3^9 \bmod 9$

b. Do you notice a pattern when the modulus is prime? If so, what did you find?

c. Use inductive reasoning to make a conjecture about the 3^p modulo p, when the modulus is prime.

Conjecture: if the modulus is prime, then... . (write a full statement, including this condition)

d. Problem 2 considered $2^p \bmod p$, and this problem $3^p \bmod p$. Suppose you found a similar result with $4^p \bmod p$, $5^p \bmod p$, $6^p \bmod p$, and so on. What conjecture might you make about $a^p \bmod p$ where a is any integer not divisible by p, and p is a prime number?

e. Prove Fermat's Little Theorem, that if a and p are relatively prime, then $a^{p-1} = 1 \bmod p$.

(You may consult any other references or use office hours.)

Postscript

If students have strong interest in modular arithmetic and access to software such as Mathematica to make computations involving large numbers (e.g., more than 10 digits), a recommended resource is Chapter 18 of Joseph Silverman's *A Friendly Introduction to Number Theory*, which will teach students how to use the RSA cryptosystem to send secret messages. This encryption system rests on modular arithmetic and properties of prime numbers. In particular, it takes advantage of Euler's formula (using Euler's Phi Function), an extension of Fermat's Little Theorem.

III.4 Reading Assignments and Discussion Questions

Reading assignments, postings on the electronic bulletin board and in-class discussions address the first two overarching goals (Section II.1) and corresponding learning outcomes:

W: Women Mathematicians in History

To examine the lives and contributions of women mathematicians from the 4th to the 20th centuries

G: Current Gender Issues in Mathematics Education, Achievement & Participation
To investigate current gender issues related to women's skills and participation in mathematics from elementary school through graduate school and their participation in math-related careers

Students will be able to

W1. Describe the life and work of 9 women mathematicians and at least one contemporary woman mathematician
W2. Synthesize from these women's biographies common experiences/obstacles faced by women who wished to participate in mathematics and identify factors that enabled their success
G1. Discuss the current situation in the United States regarding women's participation and achievement in mathematics in K-12, higher education, and industry
G2. Read critically articles in journals and newspapers dealing with gender issues in mathematics or science education

Following is a selection of the reading assignments and prompts used to address these outcomes. The course met twice a week for a total of about 28 class meetings, so the class number is included to indicate the approximate timing of the reading and discussion or posting.

Class #1
Of the five background articles required for the first paper (see Section III.6.1), read: Simon, Marilyn. (2000). "The evolving role of women in mathematics." *Mathematics Teacher. 93*(9), 782-786 and Halpern et al. (2007) available at http://www.sciam.com/article.cfm?id=sex-math-and-scientific-achievement] We will be discussing these papers in small groups, so note the major ideas in each. For Halpern (2007), questions to be prepared to discuss are: Who is Larry Summers, and why is he the lead-in to the article? What two misconceptions are behind the interpretation that women lack innate math/science ability? What gender differences in cognitive ability does the article cite? How does it substantiate those differences? What factors might influence women not to participate in math related careers? What might affect the trajectory of a woman's career in higher education? What is the major thesis of this article? How well is it supported? Other main points of this article? How supported? Was anything surprising to you?

Class #2
Read the following about Hypatia: (1) pages 9-10 in Math Equals, (2) a transcript of a radio show called Ockham' s Razor on Hypatia of Alexandria from August 3, 1997 found at http://www.abc.net.au/rn/science/ockham/or030897.htm, (3) the article Who's Hypatia? Whose Hypatia do you mean? from April 2009 issue of Math Horizons. Then, go to the Hypatia thread in the discussion board and discuss at least one of the following in your post (try to pick a new topic or something not already previously posted): the type of education that Hypatia had, what gave her the access to that education, how (un)usual it was for a women of her time to have this sort of education, what contributions she made to mathematics or to other fields. Note what source you are relying on for this information. Everyone include a comment about something you found surprising or unexpected in her life or work.

Class #5
Learn about our next woman mathematician, Emilie du Chatelet, by reading pages 29-35 in *Math Equals* and webpage from a PBS special (Nova show Einstein's Big Idea, October 2005) that talks about Châtelet's contribution to the development of Einstein's $E=mc^2$ equation at: http://www.pbs.org/wgbh/nova/einstein/ance-sq.html

Then answer all the following questions for yourself, and post on the discussion board your answers to one or more of the following questions (try to say something different than what was previously posted):

- How did Emilie have access to an education, especially a mathematical one?
- What type of participation did she have in the world of mathematics?
- Were there any surprises or unusual aspects to her life?
- What similarities/differences were there between her life/education and that of Hypatia?

NOTE: These questions about access to education, type of participation and similarities/differences to all previous women became standard for each new woman mathematician. Beginning with Emilie du Chatelet we also began to ask these additional questions:

- What sort of a mathematical topics did she work on and what sort of career did she have, if any?
- How did she handle her family responsibilities?
- Were there any surprises or new developments for women in mathematics showing up in her life?

The biographical readings and postings occurred at least every two weeks.

Class #8
Read the introduction (pp. xv11-xxi) in Claudia Henrion’s "Women in Mathematics." Post on the discussion board one new idea, insight, question or issue that this reading raises for you relative to the situation for women in mathematics.

Class #12
Read the chapter entitled Newtonian Women from the book *The Newtonian Moment: Issac Newton and the Making of Modern Culture* by Feingold. The chapter contains information about learned women contemporary with du Chatelet and Agnesi (and discusses both of them) and about children's science in the 17th-18th century. It also shows some ribald satirical cartoons of women studying science, indicative of the sentiment held by the drawers of the cartoons (and many others) that educating women (especially in math and science) was generally considered a bad idea. Post on the discussion board, one or two new insights you gained from your reading to any of the following: Emilie du Chatelet, Maria Agnesi, the Enlightenment, prevailing or changing attitudes toward women's participation in science/math in the 18th century.

Class #14
We recently read the Introduction to Henrion's book. There we encountered the idea that the practices and culture of the mathematics "community" might present another difficulty for women's achievement in mathematics. Henrion explores this idea in her essay, "Rugged Individualism and the Mathematical Marlboro Man," on pages 1-25 and then she presents two biographies of modern women mathematicians whose personal

stories enrich her essay. Everyone read pages 1-25 and if your last name begins with A-J, read Karen Uhlenbeck's personal story or if your last name begins with K-Z, read Marion Pour-El's story (both in Henrion's book). Post on the discussion board what you learned about mathematics being an individual or a community enterprise. Include something from the essay and something from your person's bio in your comment. Also note the one thing that was most surprising or unexpected or striking to you.

Class #19
Read pages 109-118 in Henrion. Post on the discussion board: What did you learn about the idea of mathematics being a young man's game? What did you find striking or surprising? How do you see this connecting to previous discussions or to the lives/work of our historical women?

Class #21
Read the excerpt on mathematics and marriage (pages 66-76) in "Change is Possible" by Pat Kenschaft. What does this reading tell us about the situation for women in mathematics relative to (a) marriage and (b) professional/career participation (married or single) during the latter half of the 20th century? Write down and bring to class 3 main points you'd like to make during class discussion (these may be collected).

Class #26
Read pages 131 to 146 of *Change is Possible* by Pat Kenschaft, which describes the beginnings of the Association for Women in Mathematics, an organization that has provided tremendous support for women in mathematics for the past 40 years, as both of your instructors can attest. Post on two or more of the following questions regarding the founding of AWM: Why was AWM needed? Of the many things that the first AWM President attempted, what stands out the most to you? why? What is your reaction to the vignettes of women mathematicians lives that are included (Everywoman on pages, 133-4; Life of a mathematical couple, pages 140-142)? How important did the AWM Newsletter appear to be? Take a look at the current webpage for the Association for Women in Mathematics (www.awm-**math**.org/). What services or programs does it offer today? Were you able to make any connections between this reading and any of the themes we have been discussing? Some of you will be able to connect this reading to your project, if so, comment on that.

III.5 Study Guides for the Quizzes

As indicated in Section II.1, two of the overall course goals are:

> *W: Women Mathematicians in History*
> To examine the lives and contributions of women mathematicians from the 4^{th} to the 20^{th} centuries
> *M: Mathematical Reasoning and Communication*
> To provide students an opportunity to experience "doing mathematics" in a supportive and cooperative environment and to encourage students to be more aware of their own mathematical thinking

The quizzes, which comprise 20% of the course grade, are designed as the primary assessment of these goals and the corresponding more specific learning outcomes:

Students will be able to

W1. Describe the life and work of 9 women mathematicians and at least one contemporary woman mathematician

W2. Synthesize from these women's biographies common experiences/obstacles faced by women who wished to participate in mathematics and identify factors that enabled their success

M1. Make and investigate mathematical conjectures, develop arguments in support of or counterexamples for those conjectures

M2. Communicate their mathematical thinking clearly to others (peers and teachers) using mathematical language

M3. Analyze and evaluate the mathematical thinking of others

M4. Recognize and appreciate: mathematics as a study of patterns; the critical and distinct roles of inductive and deductive reasoning in developing (new) mathematics; the utility of multiple representations for a single mathematical concept

Every two or three weeks students had quizzes that covered the following aspects of the course material: biographical information on the nine women mathematicians, mathematical ideas related to their work, and, to a lesser extent, information about gender-related concerns in mathematics education, achievement and participation. Because the course drew from two texts, a variety of additional readings, lectures, and mathematics worksheets, fairly detailed study guides were provided for each of the quizzes. These study guides, which follow, provide one perspective on the course material as it was presented in 2010.

Study Guide for Quiz #1

Keep these underlying mathematical principles in mind always as the course progresses -- What is mathematics and how is mathematical knowledge created? What are the roles of inductive and deductive reasoning? Are there different representations for the same mathematical question?

Gender Issues: Why are so few women mathematicians known? What are possible explanations for why women do not participate in math-related careers to the extent that men do? Why should this underrepresentation of women in math-related fields be a matter of concern? What might remedy this situation?

Underlying Mathematical Principles: What is mathematics and how is mathematical knowledge created? Are there different representations for the same mathematical question? Explain what is meant by inductive reasoning, by deductive reasoning? What is each best for? Do both methods always yield valid conclusions? Explain.

For Hypatia: What sort of education did Hypatia have? What allowed her to obtain this education? What topics in mathematics did she work on?

Mathematical Topics:

Name the three conic sections. Describe how to obtain the three conic sections by intersecting a cone with a plane. Describe how to obtain the ellipse and the hyperbola as a set of points satisfying certain distance properties. Be able to use concentric circles to draw an ellipse and hyperbola with specified distance properties (for example, draw the ellipse satisfying a distance sum of 10, or the hyperbola with a distance difference of 5). Give examples of applications or occurrences of each conic section in modern daily life. How many of these applications were known in Hypatia's time?

Define and give examples of a prime number, a composite number, and even number, an odd number. Why do mathematicians not allow 1 to be called a prime? (Hint: What does the Fundamental Theorem of Arithmetic say?) Discuss whether it is possible for the sum of two primes to be prime.

Be able to find all primes less than 200 by the Sieve of Eratosthenes.

Explain why in any factorization of a number less than n^2, at least one of the factors must be less than n.

What is meant by square numbers, rectangular numbers, triangular numbers? Be able to provide numerical, geometric and algebraic descriptions of each.

Give different definitions of "prime number." Give a geometric definition of prime number.

Be able to give algebraic proofs and picture proofs for number magic games.

Study Guide for Quiz #2

Keep these in mind always as the course progresses -- Underlying Mathematical Principles: What is mathematics and how is mathematical knowledge created? What are the roles of inductive and deductive reasoning in creating new mathematical knowledge? What kinds of different representations exist for the same mathematical question or phenomenon?

For Emilie du Chatelet and Sophie Germain: In what century and country did they live/work? What was going on in their world at the time (du Chatelet, the Enlightenment; Germain, the American, then the French, Revolution) and what was the impact of each period relative to world views or the daily life of these women? What sort of education did each woman have? What allowed each to obtain this education? What topics in mathematics did each work on? What features of each woman's life were similar to one another or to Hypatia? What differences where there in their lives?

Emilie translated and commented on Newton's *Principia*, bringing it to greater recognition on the Continent. What two completely different phenomena (both studied inductively, by whom?) did Newton's laws explain (deductively)? [See *Math Equals* pages 36-37.]

Mathematical Topics: Be able to state the definition of a function. [Answer: A function is a rule that associates to each input element in a first set (called the domain) a unique output element in a second set (called the codomain). The set of all possible outputs is called the range.] Using patterns found in data in a function machine, be able find general formulas for the rule used to define the function machine. What kind of reasoning is this? Be able to match graphs to functions describing relationships, and to identify the variables on each axis. Be able to recognize when relationships are or are not functions and explain why. List at least 3 different ways we have represented functions and give an example of each, for example graphical, numerical, algebraic or other formulas for the function rule, describing the function rule in words, or …

Give a geometric and an algebraic statement of the Pythagorean Theorem? Which could Hypatia/du Chatelet/Germain have known? Why?

<u>Previous topics that might be asked on this quiz:</u>

What is meant by square numbers, rectangular numbers, triangular numbers? Be able to provide numerical, geometric and algebraic descriptions of each.

Can you give a geometric definition of prime number?

What is Larry Summers doing now?

Study Guide for Quiz #3

Keep these in mind always as the course progresses -- Underlying Mathematical Principles: What is mathematics and how is mathematical knowledge created? What are the roles of inductive and deductive reasoning in creating new mathematical knowledge? What kinds of different representations exist for the same mathematical question or phenomenon?

<u>For Emilie du Chatelet and Maria Agnesi:</u> In what century and country did they live/work? What was going on in their worlds at the time. What sort of education did each woman have? What allowed each to obtain this education? What topics in mathematics did each work on? What features of each woman's life were similar to one another? What differences were there in their lives? How were they recognized for their work? In what ways did their lives exemplify or deviate from the tensions faced by Newtonian women interested in mathematics and science? What was one very unusual source of historical information about the personal library of Maria Agnesi?

<u>Mathematical Topics</u>:

- Be able to define the concepts of multiple, divides and Germain prime. (A nonzero integer a **divides** and integer b if we can find an integer k such that b = ka; An integer b is a **multiple** of an integer a if we can find an integer k such that b = ka; A prime number p is a **Sophie Germain prime** if 2p + 1 is also prime.) Sample question: what does it mean to say 3 does not divide x? Answer: x has a remainder of 1 or 2 when divided by 3, that is, x = 3k+1 or 3k+2 for some integer k.
- Be able to work with clock (modular) arithmetic (topic taken from Sophie Germain's work). What application does this mathematics have in modern day life? Sample questions: Solve for x: 6/10 = x(mod 12); List two positive and two negative integers congruent to 5 mod 12; Check a check-digit problem.
- Be able to describe two different interpretations for division (partitive and subtractive) and to write word problems for each interpretation. When dividing by a fraction only one of these interpretations can be used. Which one? Why?

- Find a point on the Witch of Agnesi "mechanically" and show the general shape of the graph. Given the equation of the graph be able to derive various features (symmetry, intercepts or lack thereof, asymptotes, etc.).
- Give the definition of a rational function and be able to use precalculus-level mathematics to determine properties of its graph.
- Apply the What-if-not strategy to generate new mathematical questions

<u>Previous topics that might be asked on this quiz:</u>

Give a geometric and an algebraic statement of the Pythagorean Theorem? Which could Hypatia/du Chatelet/Germain/Agnesi have known? Why?

Study Guide for Quiz #4

Keep these in mind always as the course progresses -- Underlying Mathematical Principles: What is mathematics and how is mathematical knowledge created? What are the roles of inductive and deductive reasoning in creating new mathematical knowledge? What kinds of different representations exist for the same mathematical question or phenomenon?

<u>For Mary Somerville and Ada Lovelace:</u> In what century and country did they live/work? What was going on in their worlds at the time." What sort of education did each woman have? What allowed each to obtain this education? What topics in mathematics did each work on? What features of each woman's life were similar to one another? What differences were there in their lives? How did they each balance the demands of career and family? What sort of professional status or position was each able to obtain (and how easily)? In what ways were they recognized for their work?

<u>Mathematical Topics</u>:

- Be able to use the pendulum equation to determine the relationships between the length of the string and the length of time (period) it takes to complete one swing and gravity. Describe the experiment that used this equation and information about gravity to determine the shape of the earth.

- Describe how to generate a cycloid, an epicycloid, a hypocycloid. Be able to determine how many "loops" or "scallops" the latter two will have if you know the ratio of the diameters of the two circles.

- What is the relationship between cycloids, pendulums and clocks?

- What did Galileo try to determine about the cycloid and how did he do it? What did he think this curve should be used for? What is the area under one arch of the cycloid? Name and describe the underlying method used by Roberval to get this answer. The mathematical work on the cycloid pre-dated Mary Somerville's life by a century and Galileo lived about 2 centuries before Mary, so why is the cycloid is discussed along with Mary Somerville (see *Math Equals* p. 94)?

- Be able to use difference tables to find function values for functions that are polynomials. Know the relationship between the degree of a polynomial function and the level of the first constant difference in its difference table. Be able to find the formula for a second degree polynomial given its difference table. Be able to show that for any first/second degree polynomial the first/second difference is always constant and determine what that constant is in terms of the polynomials coefficients. What are applications of difference tables?

Previous topics that might be asked on this quiz:

Give a geometric and an algebraic statement of the Pythagorean Theorem? Which could Hypatia/du Chatelet/Germain/Agnesi have known? Why?

Study Guide for Quiz #5

Keep these in mind always as the course progresses -- Underlying Mathematical Principles: What is mathematics and how is mathematical knowledge created? What are the roles of inductive and deductive reasoning in creating new mathematical knowledge? What kinds of different representations exist for the same mathematical question or phenomenon?

For Grace Chisholm Young and Sonya Kovalevskaya: In what century and country did they live/work? What was going on in their worlds at the time? What sort of education did each woman have? What allowed each to obtain this education? What topics in mathematics did each work on? What features of each woman's life were similar to one another? What differences were there in their lives? How did they each balance the demands of career and family? What sort of professional status or position was each able to obtain (and how easily)? In what ways were they recognized for their work?

For contemporary women: Describe the challenges and obstacles that 20th century women in mathematics faced relative to professional/career participation. Describe responses to those challenges. Include at least 3 specific examples. (See pages 131-146 of *Change is Possible* by Pat Kenschaft and "Is mathematics a young man's game?" essay, pages 109-120 in *Women in Mathematics: The Addition of A Difference* by Claudia Henrion.)

Mathematical Topics:

- Be able to provide a description of and examples of **simple** (no holes) and **non-simple** (can have holes) polyhedra. Recall that a polyhedron is a three-dimensional surface bounded by polygons. Which can be deformed into spheres?
- Be able to define the **Euler Characteristic** and determine it for a given polyhedron.
- Be able to describe what an 'invariant' is in mathematics and give an example of one other than the Euler Characteristic
- Be able to state **Euler's Formula** ($V + F – E = 2$ for **simple** polyhedra) and provide a sketch of the proof.
- Be able to define and give examples of **sequences** and **series.** Be able to define and recognize **geometric series,** give the nth term, the sum and the partial sum. Find sums of geometric series numerically, algebraically, and geometrically (the last only for special cases already considered). Apply

geometric series to computing the value of repeating decimals. Explain how Zeno's paradox leads to a geometric series.

Previous topics that might be asked on this quiz:

Give a geometric and an algebraic statement of the Pythagorean Theorem? Which could Hypatia/du Chatelet/Germain/Agnesi have known? Why?

Be able to define the concepts of inductive reasoning and deductive, explain their roles in the discovery/development of new mathematics, and give examples of each applied to the same problem or question.

Study Guide for Quiz #6

Keep these in mind always as the course progresses -- Underlying Mathematical Principles: What is mathematics and how is mathematical knowledge created? What are the roles of inductive and deductive reasoning in creating new mathematical knowledge? What kinds of different representations exist for the same mathematical question or phenomenon?

For Emmy Noether: In what century and country did she live/work? What was going on in her world at the time? What sort of education did she have? What allowed her to obtain this education? What topics in mathematics did she work on? What features of her life were similar to and different from the earlier women? How did she balance the demands of career and family? What sort of professional status or position was she able to obtain (and how easily)? In what ways was she recognized for her work?

Mathematical Topics:

- Define the notion of a **group** (a set of objects together with an operation for combining them that satisfies the properties of: closure, associativity, identity, and inverse. A group operation may or may not be commutative.)

- Be able to describe and recognize the properties of identity, closure, commutativity from a Cayley table.

- Be able to describe the property of associativity.

- Be able to give examples of sets and operations that do and do not have the properties of closure, associativity, identity, inverses, and/or commutativity.

Previous topics that might be asked on this quiz:

Be able to:

- Define the concepts of inductive reasoning and deductive reasoning, explain their roles in the discovery/development of new mathematics, and give examples of each applied to the same problem or question.

- Discuss the value of multiple representations in mathematics. Be able to give several examples of mathematical questions or phenomena from our class for which we presented multiple representations.

- Describe the challenges and obstacles that 20th century women in mathematics faced relative to professional/career participation. Describe responses to those challenges. Include at least 3 specific examples. (See pages 131-146 of *Change is Possible* by Pat Kenschaft and "Is Mathematics a young man's game?" essay, pages 109-120 in *Women in Mathematics: The Addition of A Difference* by Claudia Henrion.)

III.6 Reports, Projects, & Final Reflective Writing Assignments

III.6.1 First Report Assignment: The Women-and-Mathematics Situation

As the indicated in Section II.1 one of the overall course goals is:

> *G: Current Gender Issues in Mathematics Education, Achievement & Participation*
>
> To investigate current gender issues related to women's skills and participation in mathematics from elementary school through graduate school and their participation in math-related careers

The first major report, a paper discussing "The Women-and-Mathematics Situation," is the primary assessment for this goal and the corresponding more specific learning outcomes:

> G1. Discuss the current situation in the United States regarding women's participation and achievement in mathematics in K-12, higher education, and industry
>
> G2. Read critically articles in journals and newspapers dealing with gender issues in mathematics or science education

These outcomes are also addressed again in the first quiz, and in the final reflective writing.

The course opens with portions of the first three classes devoted to readings, lectures, in-class discussions, and required postings to an electronic bulletin board about gender-related concerns in mathematics education, achievement and participation dating back to the 1970s. This material is to be synthesized in a written report that is due during the fourth week of the semester. An annotated bibliography of the required readings is due during the third week of the semester. The actual assignment, as given in Fall 2010, and the rubric for grading the paper follow. One of the challenges in teaching this course is to update readings and presentations on the continually evolving topic of women and mathematics.

<u>Directions:</u> This paper on gender issues in mathematics education, achievement and career participation, "The Women-and-Mathematics Situation," is to be based on class presentations and discussion and, at a minimum, on the five required readings listed below. It must: (1) address the following topics, (2) include an annotated bibliography (see sample annotated entry provided above for the Jackson article) and (3) end with the indicated Appendix, as described below.

Topics: What is the current situation with regard to female participation and achievement in mathematics (in school, on standardized tests, and in math-related careers) in the United States? How has it evolved over the last 30 or 40 years (or longer)? What problems remain? How might they be addressed?

Bibliography: Be sure to include all 5 required papers in your bibliography and to cite any additional sources you use. Your bibliography should be annotated, that is, each entry should be followed by a short summary. We have annotated the second recommended reading above as a model.

Appendix: At the end of your paper, in an appendix, discuss how any of these gender/math issues may have touched you personally or how might they in the future.

Length Expectations: To properly address the essential topics, this paper is anticipated to be 4-6 pages in length, double spaced, plus appendix (1-2 pages) & bibliography.

Important Note: Being able to articulate and support your position on the issues addressed in this paper is integral to several course goals/objectives and you may be asked to do so again on the first mid-term and/or later projects.

Five Required Readings:

1. Begley, S. (2009). The Math Gender Gap Explained. *Newsweek blog*, *"The Human Condition."* [Accessed online 12-30-11: http://blog.newsweek.com/blogs/thehumancondition/archive/2009/06/01/sharon-begley-the-math-gender-gap-explained.aspx]

2. Gordon, Carolyn & Keyfitz, Barbara. (2004, August). Women in academia: Are we asking the right questions" *Notices Amer. Math. Soc. 51*(7), 784-786. [Available online: http://www.ams.org/notices/]

3. Halpern, D., Benbow, C., Geary, D., Gur, R., Hyde, J., & Gernsbache, M. (2007, Fall). Sex, math and scientific achievement: Why do men dominate the fields of science, engineering and mathematics? *Scientific American.* [Accessed online 12-30-11 http://www.sciam.com/article.cfm?id=sex-math-and-scientific-achievement]

4. Hyde, J., Lindberg, S., Linn, M., Ellis, A., & Williams, C. (2008, July). "Gender Similarities Characterize Math Performance." *Science. 321*(5888), 494-495. See http://www.sciencemag.org/content/vol321/issue5888/

5. Simon, Marilyn. (2000, December). "The evolving role of women in mathematics." *Mathematics Teacher. 93*(9), 782-786.

Additional Recommended Readings[1]:

[1] For data on minorities: Medina, Herbert. “Doctorate Degrees in Mathematics Earned by Blacks, Hispanics/ Latinos, and Native Americans: A Look at the Numbers." *Notices Amer. Math. Soc*. 51(7) August 2004, 772-75.

1. Green, Judy. (2001, November). How many women mathematicians can you name? *Math Horizons.* 9, 9-14.

2. Jackson, Allyn. (2004, August). Has the women-in mathematics problem been solved? *Notices American Mathematical Soc*iety, *51*(7), 776-783. [Available online: http://www.ams.org/notices/] This article identifies which Ph.D. granting institutions produce the most women doctorates by number and by percentage. It examines features of these programs that might be responsible for their success. Despite encouraging signs, the article also identifies problem areas, asks why the once urgent problem of the dearth of women at top institutions is no longer an issue, and argues that special programs and organizations are still needed.

3. Andreescu, T., Gallian, J., Kane, J., & Mertz, J. (2008). Cross-Cultural Analysis of Students with Exceptional Talent in Mathematical Problem Solving. *Notices of the American Mathematical Society, 55*(10), 1248-1260. [Available online: http://www.ams.org/notices/200810/tx081001248p.pdf]

Grading Rubric for Women-and-Mathematics Situation Report

The possible points for each item below is indicated in parentheses. The total possible is 100 points.

Topics to be Addressed (50 points distributed as indicated below)

- Current situation with regard to female participation/achievement in mathematics in the US--in school (15 points)
- Current situation with regard to female participation/achievement in mathematics in the US--in math-related careers (15 points)
- How have these situations evolved over the last 30 or 40 years (or longer)? (10 points)
- What problems remain? How might they be addressed? (10 points)

Overall Coherence and Logical Flow (10 points)

The first paragraph or two of an academic paper should provide a context for the paper AND also help the reader by previewing the content and structure of the paper. See Halpern et al. (2007), paragraph 3, for an example of "previewing content."

Style [precise, effective word choice/phrasing; varied coherent sentences; minimal repetition] (10 points)

Reading your essay out loud is one way to find out how smoothly and coherently it reads. If you stumble over a phrase or sentence it might need fixing. Take note if the same word (or similar wording) appears 3 or more times within 2 or 3 sentences. Fix this by varying the wording or by tightening up the writing – maybe you are just repeating yourself. Watch out for your pronoun usage. Is it clear what "it" or "this" refers to, or are you just being a bit lazy and using a pronoun rather than striving for an accurate phrase? Ask a friend to read your paper and tell you what is not clear.

Technical Control [spelling, grammar, punctuation, and presentation] (10 points)

Proof-read, proof-read, proof-read! Watch for singular-plural mismatches. Sometimes students write very long sentences with several clauses but fail to use commas to help the reader parse through the sentence so it's hard to *understand the intended meaning. (Would a comma or two make it easier to read the last sentence?)*

Annotated Bibliography (10 points)

Paper draws on all 5 required readings. Supporting statements and quotes given in the body of the paper are cited (any standard style – MLA, APA, etc is acceptable, but APA preferred and http://owl.english.purdue.edu/owl/resource/560/01/ is an excellent resource for APA style), all references are included in the bibliography, and all references are annotated (an example of an annotated entry was given in the directions to the assignment).

Appendix (10 points)

Include in an appendix a personal reflection how any of these issues might connect to you now or in the future.

III.6.2 Research Project

Both the math and the honors versions of the course include as an overarching goal to have students undertake an interdisciplinary research project (Section II.1). For students registered under mathematics or under honors, the corresponding learning outcomes are, respectively, to

R1. Demonstrate the ability to conduct an interdisciplinary research project

or

H1. Demonstrate the ability to make connections between disparate disciplines.

Students are provided with a list of possible topics and an initial set of references. They also have the option of proposing their own topic. Students are advised to select a topic they find particularly interesting and they submit their first, second and third choices. Based on their requests, the instructor makes the assignments. Topics fell into the broad categories of mathematical investigations, issues for future teachers, gender equity concerns in science and mathematics, historical investigations, special issues for minority women, scientific couples, and Nobel prize winners. One of the challenges in teaching this course is keeping the list of potential topics fresh and the suggested references up to date. The list of topics provided in 2010 appears immediately below. The grading rubric for the research project appears after the list of topics.

Research Project Topics for Women and Mathematics (Fall 2010)

Note: [MM/prerequisite] indicates the topic is most appropriate for Math Major or Minor/and has a recommended MATH course prerequisite.

Mathematical Investigations

Sophie Germain and her eponymous primes Delve more deeply into Germain's life and her work using chapter 11 (Sophie Germain and the Attack on Fermat's Last Problem) in Steven Krantz's *An Episodic History of Mathematics: Mathematical Culture Through Problem Solving (2010)* published by the MAA. Follow up with the references given at the end of the chapter, explicate the math in the chapter and do some of the early exercises at the end of the chapter, then challenge yourself with one of the projects outlined at the end of the exercises. [MM/Group Theory]

Maria Agnesi and her Calculus book Maria Agnesi's *Analytical Institutions* is available on-line via google books at http://www.archive.org/details/analyticalinsti00masegoog Recall that her project of translating Newton's *Principia* into Italian was initiated as a text for her younger siblings. With what mathematical topics does her book begin? Work some of her max/min problems (See 2 suggested problems in Katz, *A History of Mathematics 3rd edition* p. 638, #30 and 32. Approach these problems from multiple perspectives.)

Geometric Constructions Using Paper Folding Techniques #1 Learn how to use paper folding to illustrate geometric constructions such as similar triangles, equilateral triangles, parabolas, trisecting an angle, and dividing a length into n equal parts in Thomas Hull's book, "Project Origami: Activities for Exploring Mathematics." Some of these projects are applicable to middle-school and high-school classrooms!

Geometric Constructions Using Paper Folding Techniques #2 Use Grace Young's book to direct your investigations. Find connections to the K-12 classroom. Internet search: Patty paper geometry, Key Curriculum Press, NCTM and Math Forum.

The arithmetic-geometric mean Explain the arithmetic-geometric mean, derive some of its properties, and discuss its applications. Trace its connections to well-known and obscure mathematicians. Among the latter falls John Landen the publisher of the *Ladies' Diary*. Reference: Almkvist, G. and Berndt, B. "Gauss, Landen, Ramanujan, the Arithmetic-Geometric Mean, Pi, and the *Ladies Diary*." *Amer. Math. Monthly* 95(7), 585-608. [MM/Real Variables]

What was the mathematics in The Ladies Diary? Begin with Teri Perl's "The Ladies' Diary or Woman's Almanack, 1704-1841," *Historia Mathematica,* 6(1979) 36-53 to investigate the mathematical content. Using the references in this article find out more about the typical education and societal norms of the time period.

Who was Miss Mullikin and what is her mathematical nautilus? Read *Who Was Miss Mullikin?* by Thomas L. Bartlow and David E. Zitarelli, published in the February 2009 American Mathematical Monthly. Then develop your understanding of the mathematical concepts (connected sets in one and two dimensions) and present the results that appeared in the Anna Mullikin's 1922 dissertation. Who was her thesis advisor? Why is he a controversial figure in the world of mathematics? Find additional print resources referred to on these useful internet sites:
http://mathdl.maa.org/mathDL/46/?pa=content&sa=viewDocument&nodeId=1582&pf=1
http://mathdemos.gcsu.edu/mathdemos/mullikin/mullikin.html [MM/Real Variables]

Solving the 3 impossibility problems of antiquity: Squaring the circle, trisecting an angle and duplicating the cube. A significant amount of new mathematics was motivated by these three problems, which are impossible to solve with a finite number of constructions using only a straightedge and compass. The various approaches involve conic sections (350 BCE), mechanical devices (230 BCE) and special curves (450BCE through 1659 CE). There are multiple projects possible within this topic. [MM/Precalculus and High School Geometry]

Combine crafts and mathematics. Choose either "Socks with Algebraic Structure" (group theory) or "(K)not Cables, Braids" (group theory and topology) from *Making mathematics with needlework* (2008) by belcastro and Yackel and learn how these mathematical topics appear in needlework crafts. In addition to the mathematics, you can investigate the controversy over a review of the book in the *Association for Women in Math Newsletter*. [MM/Group Theory] And you can make a historical connection between scientific women and the craft tradition using Chapter 3 of Londa Schiebinger's book *The Mind Has No Sex: Women in the origins of modern science*. Also comment on the participation of the football player Rosie Grier in needlework crafts.

Issues for Future Teachers

Math Anxiety Start with "Math-Abused Students: Are We Prepared to Teach Them?" by Greg Fiore in *The Mathematics Teacher* May 1999, 92(5). Then see Frankenstein, Marilyn. "Overcoming Math Anxiety by Learning About Learning" (Mathematics and Computer Education, v18 n3 p169-80 Fall 198) pp. 169-181. (This article states misconceptions about learning and gives suggestions for overcoming them. It would be very appropriate for a future teacher to read. It contains an excellent annotated bibliography.)

Compare and contrast two classics about math anxiety: *Overcoming Math Anxiety* by Sheila Tobias and *Mind Over Math* by Kogelman and Warren.

A fairly recent book about overcoming math anxiety which has excellent information on how to develop good mathematical study skills is *Defeating Math Anxiety* by Kitchens, published by Richard D. Irwin, Inc. 1995.

The I Hate Mathematics Book by Marilyn Burns, Little. Brown and Company, Boston, 1975. (This book is full of mathematical activities for kids and grownups. Anyone planning to teach school should know about this book.)

Math Doesn't Suck: How to Survive Middle School Math by Wonder Years' actress Danica McKellar has a provocative title. Check accuracy of claims about gender differences on the book's website and examine the mathematical explanations that appear in the book for accuracy and 'best practice." See posts on this topic from August 2007 on RUME and POD Listserves http://listserv.nd.edu/cgi-bin/wa?A2=ind0708=pod=D=16976 <http://listserv.nd.edu/cgi-bin/wa?A2=ind0708&L=pod&O=D&P=16976>
http://listserv.nd.edu/cgi-bin/wa?A1=ind0708=pod#119 <http://listserv.nd.edu/cgi-bin/wa?A1=ind0708&L=pod#119>
Compare the mathematical explanations in her 2 more recent books *Kiss my Math* and *Hot X: Algebra exposed*

Stereotype Threat Explore this topic at greater depth. See Claude Steele's groundbreaking work and more recently the 2007 paper "Stereotype Threat and Working Memory: Mechanisms, Alleviation, and Spill Over" in the *Journal of Experimental Psychology: General* by Robert Rydell and Allen McConnell.

Various Aspects of Math/Science Gender Issues Base your project on one of the following resources or ideas, using the suggested aspect or an idea of your own.

Gender differences in the careers of academic scientists and engineers What are the gender differences in career outcomes and what influence does family characteristics (i.e., married? children?) have on the outcome? What did *Chronicle of Higher Ed* publish about this topic recently? What implications does this have? Personalize this report by interviewing some faculty. See:
http://www.nsf.gov/statistics/nsf04323

Mathematics and Gender from a Feminist Perspective Start with: "Gender and Mathematics from a Feminist Standpoint" by Damarin and "Redefining the 'Girl Problem' in Mathematics" by Campbell in *New Directions for Equity in Mathematics Education* (edited by Secada, Fennema and Adajian, Cambridge Univ. Press, 1995

How equal is the education that boys and girls receive? Start with the following:

Sadker, Myra and David Sadker. *Failing at Fairness How America's Schools Cheat Girls*. Charles Scribner's Sons. New York, 1994.

American Association of University Women (AAUW) report "How Schools Shortchange Girls" and AAUW videos *Girls Can* and *Girls in the Middle: Working to Succeed in School* [google AAUW]

Is there a boys' crisis in education? Nearly 60% of college students are women. Have girls advanced educationally at the expense of boys? Summers (2000) would have us believe so in The War Against Boys: How misguided feminism is harming our young men. What does the AAUW report say on this topic? See *Where the Girls are: the facts about gender equity in education*. URL = http://www.aauw.org/research/WhereGirlsAre.cfm (See also the short article in AAUW *Outlook* Spring/Summer 2008)

Examining and Changing Classroom Culture Start with the following: "Uncovering Bias in the Classroom - A Personal Journey" by Wickett, "Creating a Gender-Equitable Multicultural Classroom Using Feminist Pedagogy" by Jacobs, and "Know Thyself: The Evolution of an Intervention Gender-Equity Program" by Koontz in *Multicultural and Gender Equity in the Mathematics Classroom: The Gift of Diversity (1997 Yearbook)* published by NCTM.

Overlapping Concerns of Multicultural and Gender Equity Start with the following: *Multicultural and Gender Equity in the Mathematics Classroom: The Gift of Diversity (1997 Yearbook)* published by NCTM and *New Directions for Equity in Mathematics Education* (edited by Secada, Fennema and Adajian, Cambridge University Press, 1995)

Equitable Instruction for Hispanics Start with the following:

Changing the Faces of Mathematics: Perspectives on Latinos, edited by Luis Ortiz-Franco, Norma G. Hernandez, and Yolanda De La Cruz, published by NCTM 1999.

"Culturally Relevant Mathematics Teaching in a Mexican American Context" by E. Gutstein, P. Lipman, P. Hernandez, and R. de los Reyes in *J. for Research in Math. Ed*. 28(6) December 1997, 709-737.

Equitable Instruction for African Americans Start with the following:

Changing the Faces of Mathematics: Perspectives on African Americans, edited by Marilyn Strutchens, Martin L. Johnson, and William F. Tate, published by NCTM 2000.

"It doesn't add up: African American students' achievement" by Gloria Ladson-Billings in *J. for Research in Math. Ed*. 28(6) December 1997, 697-708.

Equity Concerns in Science and Mathematics

Errors in Scientific Thinking and Approaches Compare and contrast the books *The Mismeasure of Man* by Stephen J. Gould and *The Mismeasure of Women* by Carol Tavris

Feminist Science -- What is it? Does it exist? Some hold the viewpoint that women's perspectives and contributions have been different from men's in science and therefore have set new directions or made unexpected discoveries. Explore this concept:

- *Gendered Innovations in Science and Engineering* by Londa Schiebinger, Stanford University Press, 2008.
- "Is There a Female Style in Science?" *Science*, Vol. 260, April 16, 1993, pp 386-391.
- "Feminists Find Gender Everywhere in Science" Science, Vol. 260, April 16, 1993, pp 392-393.

For a more philosophical view of this question consult (Section 5 of): Anderson, Elizabeth, "Feminist Epistemology and Philosophy of Science", The Stanford Encyclopedia of Philosophy (First published Wed Aug 9, 2000; substantive revision Thu Feb 5, 2009), Edward N. Zalta (ed.), URL = http://plato.stanford.edu/entries/feminism-epistemology/

Is the situation for women in mathematics worse than in the other sciences? See the article by Paul Selvin in the March 13, 1992 issue of *Science* and the response to it in the July 17, 1992 issue of Science signed by 18 women mathematicians. Also see the special issue of the *Notices* of the American Mathematical Society (Vol. 38, no. 7) on the status of women mathematicians published in September 1991 and the most recent update in the August 2004 *Notices* "Has the Woman in Mathematics Problem Been Solved?*"* by Allyn Jackson.

What encourages (discourages) women in mathematics? Start with the following:

Winning women into mathematics by Pat Kenschaft (ed) published by The Mathematical Association of America, 1991.

Women in Mathematics and Physics: Inhibitors and Enhancers (Executive Summary, December 1, 1991), reprinted in *AWM Newsletter* vol. 22, no 3, May-June 1992, pp. 12-22. (A copy of the full report may still be available from the Center for Education of Women at University of Michigan, 330 E. Liberty Street, Ann Arbor, MI 48104-2289; 313-998-7240.)

Women and Work Start with the following:

The Second Shift: Working Parents and the Revolution at Home (New York 1989)

Check the RAND Corporation website for more recent reports than the two listed below.

Women in Nontraditional Occupations: Choice and Turnover by Linda Waite and Sue Berryman. A Rand report #R-3106-FF, March 1985.

US. Women at Work by Linda Waite. A Rand report #R-2824-RC, December 1981.

This topic would be best explored by finding updated references.

Film & TV Representations of Mathematically Talented Women What are some recent films and TV shows that depict mathematically talented women? How are they represented (positively, negatively? Stereotypically?) What do people learn about mathematically talented women from film and TV representations? Begin this project with an informal survey of your peers, and summarize what you hear in writing. Add your own perceptions. Then consult the article that appeared in the Feb 2009 issue of the *AWM Newsletter,* by Sarah Greenwald and Jill Thomley entitled *Mathematically Talented Women in Film and Television: A summary of the last five years*.

Why Physics might be the most difficult science for women to gain equity? Begin by reading *Pythagoras' Trousers* by M. Wertheim, and then look at what the American Institute of Physics has to say.

Historical Investigations

Catholic enlightenment and modern science Using *The World of Maria Agnesi, Mathematician of God* by Massimo Mazzotti (2007) as a starting point explore how Enlightenment Catholics (modified tradition in an effort to reconcile aspects of modern philosophy and science with traditional morality and theology. How are the Jesuits viewed in this book? Also see Newtonian Women in *The Newtonian Moment: Isaac Newton and the Making of Modern Culture* by Mordechai Feingold New York, Oxford Univ press 2004

Historical and Literary Perspectives on Sofia Kovalevsky Sofia (Sonia) Kovalevsky was not only a mathematicians of great renown, but an author of her own biography, *A Russian Childhood* and of a novella, *Nihilist Girl*. Read these two books (both are in the Hannon library) and consult Ann Hibner Koblitz's comprehensive biography of SK (also available at Hannon Library). Find out more about the historical period of her lifetime (post-Crimean war, 1850-1891). Present a synthesis of the forces acting on SK's generation, class and gender. How are these revealed in her novella? In her life? Why did SK abandon math for literary work in the later years of her short life? What other questions and observations do your explorations of her work and her time bring forth?

Influences of the 17th and 18th centuries on the place of women in science See *The mind has no sex* by Scheibinger (1989).

Pioneering Women in America Mathematics: The Pre-1940s PhDs. Read Green and LaDuke's book and compare and contrast the issues they identify for these women (educational opportunities, career patterns) with our historical women using both their essays and their biographical entries.

Special Issues: Minority Women, Scientific Couples, Nobel Prize Winners

Minority Women Mathematicians How many are there? Who are they? What special problems did/do they face? How are they able to overcome them?

Change is possible: Stories of women and minorities in mathematics P. Kenschaft, Amer. Math. Soc. 2005

"Double Jeopardy: Gender and Race" Chapter 5 in Claudia Henrion's *Women in Mathematics: The Addition of Difference*.

Kenschaft, Patricia. "Black Women in Mathematics in the United States," *The American Mathematical Monthly*. Vol. 88 No. 8 (October 1981), pp. 592 - 604.

"A Minority Woman's Viewpoint." By Eleanor Jones in *Winning women into mathematics* by Pat Kenschaft (ed), published by The Mathematical Association of America, 1991.

Newell, Virginia (ed) *Black Mathematicians* Ardmore, PA: Dorrance and Co., 1980.

The Mathematics Teacher published an article about Gloria Hewitt, and African American mathematician, in the last 10 years.

See also: Medina, Herbert. "Doctorate Degrees in Mathematics Earned by Blacks, Hispanics/Latinos, and Native Americans: A Look at the Numbers." *Notices Amer. Math. Soc*. 51(7) August 2004, 772-75. [Available on-line: http://www.ams.org/notices/]

Women of Color in Mathematics, Science and Engineering: A Review of the Literature from the Center for Women Policy Studies, 2000 P Street, NW, Suite 508, Washington DC 20036 (202) 872-1770.

Investigate Husband and Wife Teams *Creative Couples in the Science* edited by Pycior, Slack, and Abir-am published by Rutgers University Press 1996 looks at this issue from several perspectives. It includes articles on Nobel Prize winning couples (there are 3); on couples who began as student-instructor pairs; mutually supportive couples; couples whose relationships devolved into dissonance; and a comparison across disciplines. Extensive bibliography. See also *Association for Women in Science (AWIS) Magazine* vol. 25 no. 4 Summer 1996 devoted to Dual Career Couples. Possibilities include: (1)
Einstein and Mileva Maric (an article appeared in *Physics Today* in early August, 1994) Troemel-Ploetz, Senta. "Mileva Einstein-Maric: The Woman Who Did Einstein's Mathematics," *Women's Studies International Forum*, vol. 13., no. 5., 1990; Chapter 13 in *Creative Couples in the Sciences*; and Albert Einstein and Mileva Maric: The Love Letters, translated by Shawn Smith, Princeton, 1992. Also, see our library for recent videos. (2) The Curies. See Chapter 1 in *Creative Couples in the Sciences*; A more modern example can be found amongst astronomers, for example, the Shoemaker-Levy team.

Dual Career Couples (sometimes called the two-body problem) A faculty member with a spouse domestic partner or significant other who is also interested in an academic career faces additional challenges in finding a successful career in academia, even more so if the spouse's disciplinary area is the same as the faculty members. How many women mathematicians (scientists, or engineers) find themselves in this situation? What options are available to them, what issues do they face if hired together, what help is available to them to find positions together? Find national trends and recommendations in: (1) *Recommendations on Partner Accommodation and Dual Career Appointments* (2010), a report by AAUP (available online at http://www.aaup.org/AAUP/comm/rep/dual.htm), (2) *Dual Career Academic Couples: What Universities Need to Know* (2008), a report published by the Michelle R. Clayman Institute for Gender Research at Stanford University (available on-line at http://www.stanford.edu/group/gender/ResearchPrograms/DualCareer/) and (3) *The Two-Body Problem: Dual-Career-Couple Hiring Practices in Higher Education* (2002) by Lisa Wolf-Wendel , Susan B. Twombly, Suzanne Rice. What is recommended or common practice relative to hiring dual career couples at LMU?

Nobel Prize Winning Women (and why is there no Nobel in Mathematics?)

McGrayne, Sharon *Nobel Prize women in Science: Their Lives, Struggles and Momentous Discoveries.* Birch Lane Press, 1992/3?

Garding , Lars and Lars Hormander "Why is there no Nobel Prize in mathematics?" *Mathematical Intelligencer* vol. 7, no. 3 pp. 73-74.

Crawford, Elizabeth. *The Beginnings of the Nobel Institution*, Cambridge Univ. Press, 1984.

Grading Rubric for the Research Project

Abstract (150 words max) and Annotated Bibliography (10 points)
Supporting statements and quotes given in the body of the paper are cited (any standard style – MLA, APA, etc is acceptable, APA preferred), all references are included in the bibliography, and all references are annotated).

Appendix (10 points)
In an appendix of at least one double-spaced page in length reflect on 3 topics: how this project connects to the course content and goals; how it enhanced your understanding of mathematics, mathematics-related gender issues or both; and what you might have done to improve the overall product.

Content (50 points) *Expectations will vary to some extent with regard to the content depending on whether the paper topic is more mathematical or more focused on gender. Content will be evaluated on: The breadth and depth of content (15-20 points), the clarity (including proper use of mathematical notation and terminology, if applicable) and strength of the 'arguments' presented (10-15 points), the quality of the resources (5 points), and the addition of human interest factor (5-10 points) possibly through historical connections, fascinating examples, personal interviews, or special illustrations.*

Overall coherence and logical flow of the paper (10 points)
The first paragraph or two of an academic paper should provide a context for the paper AND also help the reader by previewing the content and structure of the paper. See Halpern et al. (2007), paragraph 3, for an example of "previewing content." Transitional sentences or even paragraphs are key to moving a 'discussion' forward logically in a way that the reader can follow. Sometimes section headings can be useful in clarifying the structure of a longer paper. At the end of the paper provide an appropriate summary and conclusion.

Style (10 points) precise, effective word choice/phrasing; varied coherent sentences; minimal repetition
Reading your essay out loud is one way to find out how smoothly and coherently it reads. If you stumble over a phrase or sentence it might need fixing. Take note if the same word (or similar wording) appears 3 or more times within 2 or 3 sentences. Fix this by varying the wording or by tightening up the writing – maybe you are just repeating yourself. Watch out for your pronoun usage. Is it clear what "it" or "this" refers to, or are you just being a bit lazy and using a pronoun rather than striving for an accurate phrase? Ask a friend to read your paper and tell you what is not clear.

Technical Control (10 points) spelling, grammar, punctuation, and presentation
Proof-read, proof-read, proof-read! Watch for singular-plural mismatches. Sometimes students write very long sentences with several clauses but fail to use commas to help the reader parse through the sentence so it's hard to understand the intended meaning. (Would a comma or two make it easier to read the last sentence?)

III.6.3 A Contemporary Woman Mathematician Poster Assignment

As indicated in Section II.1, two learning outcomes of this course relate to contemporary women mathematicians. They are:

W1. Describe the life and work of 9 women mathematicians and at least one contemporary woman mathematician
W2. Synthesize from these women's biographies common experiences/obstacles faced by women who wished to participate in mathematics and identify factors that enabled their success

The Contemporary Woman Mathematician Poster and Presentation Assignment addresses this learning outcome and serves as an assessment of it. The following materials describe the assignment and provide information and materials for creating an electronic poster. A rubric is provided for assessing both the poster and the presentation. This assignment draws upon the electronic poster tool available through MERLOT (www.merlot.org) and assumes that the instructor has her own account and has created a sample poster to be used as a template (see below).

Contemporary Women Mathematicians Poster Information

DIRECTIONS: Follow these steps to produce your poster.

#1 Using your school email address as your username, **create an account at MERLOT**: www.merlot.org
Click on "Register now!" (just below the login area on the right) and follow the prompts.

#2 **Email your user name** (the email address you used to create the account) to **insert instructor's email address** so that she can share with you through MERLOT the sample poster on Mary Ellen Rudin to use as a template. (The sample poster provided to students is available at http://contentbuilder.merlot.org/toolkit/html/snapshot.php?id=94890482346425.)

Once the instructor sends the template, you will be able to view and edit it by clicking on "Access the Content Builder" which appears on the right hand side, under the "Logged In" heading.

#3 A list of contemporary woman mathematicians and some biographical information for each follow. Selections of which woman to focus on will be made in class. You are welcome to seek additional information. For each of the following categories, decide on the content and type it into a word document to take advantage of spell-check. Proof-read too!

- Modern Woman Mathematician's Name
- Poster Author(s) Name(s)
- Early Life and Education
- Attraction to Mathematics
- Career Opportunities and Choices
- Obstacles Faced & Overcame
- Compare/Contrast to Earlier Women
- Recognitions/Honors/Firsts
- Modern Themes (Obtaining a job; Child care, if appropriate)
- SOMETHING SPECIAL (For example, Her Reflections on Mathematics)
- Bibliography (include photo sources)

#4 Copy and paste each category into the poster template. As noted above, you can view and edit the template by clicking on "Access the Content Builder" which appears on the right hand side, under the "Logged In" heading. You will replace all blue shaded areas and blue typeface. (Choose any colors you want.)

#5 When your poster is finished, select the poster (click in the square next to the file name) and click on **Give Copy To** option to share the poster with your instructor by e-mailing it to **instructor's email address**. Your posters should be shared with the instructor the day prior to your presentation.

Bibliography for 2010 Contemporary Women Mathematicians Poster

Many of these women have biographies with extensive reference lists on the Agnes Scott College "Biographies of Women Mathematicians" website:

http://www.agnesscott.edu/Lriddle/women/women.htm

You might also consult:

http://www-gap.dcs.st-and.ac.uk/~history/BiogIndex.html

Joan Birman

- Newsletter of the Association for Women in Mathematics, 26(3), May-June 1996, 5-6. [Reprinted in Complexities: Women in Mathematics, Bettye Anne Case and Anne Leggett, Editors, Princeton University Press (2005), 199-201.]
- Henrion, Claudia. Women in Mathematics: The Addition of Difference, Indiana University Press, 1997, 121-140.
- Joan Birman, "In Her Own Words," AMS Notices, Vol. 38, No. 7, Sept. 1991, pp. 702-706. (Reprinted at AWM web site)

Lenore Blum

- Perl, Teri. Women and Numbers: Lives of Women Mathematicians. Wide World Publishing, 1993, 77-93.
- Perl, Teri. "Lenore Blum," *Notable Women in Mathematics: A Biographical Dictionary,* Charlene Morrow and Teri Perl, Editors, Greenwood Press, 1998, 11-16.
- Henrion, Claudia. *Women in Mathematics: The Addition of Difference*, Indiana University Press, 1997, 145-163.
- "Professor tries to instill passion for math, science". August 21, 2005, article from the Pittsburgh Post-Gazette about Blum's work to encourage girls to consider careers in math and science.
- Blum, Lenore. "Women in Mathematics: An International Perspective, Eight Years Later," The Mathematical Intelligencer, 9(2) 1987, 28-32.

Fan Chung

- Don Albers. Making Connections: A Profile of Fan Chung, Math Horizons, September 1995, 14-18.
- Brunner, Regina. "Fan King Chung," *Notable Women in Mathematics: A Biographical Dictionary,* Charlene Morrow and Teri Perl, Editors, Greenwood Press, 1998, 29-34.
- Henrion, Claudia. *Women in Mathematics: The Addition of Difference*, Indiana University Press, 1997, 97-108.

Ingrid Daubechies

- Von Baeyer, Christian. "Wave of the future," Discover, May 1995, 68-74.
- Kort, Edith. "Ingrid Daubechies," *Notable Women in Mathematics: A Biographical Dictionary,* Charlene Morrow and Teri Perl, Editors, Greenwood Press, 1998, 34-38.
- Daubechies, Ingrid. "Thought Problems," an autobiographical essay in *Complexities: Women in Mathematics*, Bettye Anne Case and Anne Leggett, Editors, Princeton University Press (2005), 358-361.
- Haunsperger, Deanna and Stephen Kennedy. "Coal Miner's Daughter," Math Horizons, Mathematical Association of America, April 2000, 5-9 and 28-30.
- "Ingrid Daubechies Receives NAS Award in Mathematics," Notices of the American Mathematical Society, May 2000, p571.

Etta Falconer

- Kenschaft, Patricia. "Black Women in Mathematics in the United States," American Mathematical Monthly, 88(8) 1981, 592-604.
- Parker, Ulrica Wilson. "Etta Falconer," *Notable Women in Mathematics: A Biographical Dictionary,* Charlene Morrow and Teri Perl, Editors, Greenwood Press, 1998, 43-47.
- "Hay Award for Contributions to Mathematics Education," Notices of the AMS, May 1995, 563-564.
- AWM Newsletter, Vol. 25, No. 2, March-April 1995, 3-4.

Evelyn Boyd Granville

- Kenschaft, P. "Black Women in Mathematics in the United States," The American Mathematical Monthly, 88(8) October 1981, 592-604.
- Giles-Giron, Jacqueline. "Black Pioneers in Mathematics: Brown [sic], Granville, Cox, Claytor, and Blackwell," American Mathematical Society Focus, January-February 1991, 18.
- Gonzalez, Hannia. "Evelyn Boyd Granville," *Notable Women in Mathematics: A Biographical Dictionary,* Charlene Morrow and Teri Perl, Editors, Greenwood Press, 1998, 66-71.
- Granville, Evelyn Boyd. "Looking Back...Looking Ahead," from the Olga Taussky Todd Celebration of Careers for Women In Mathematics, reprinted in *Complexities: Women in Mathematics*, Bettye Anne Case and Anne Leggett, Editors, Princeton University Press (2005), 272-281.
- Perl, T. *Women and Numbers: Lives of Mathematicians plus Discovery Activities*. World Wide Publishing/Tetra, 1993, 95-107.

Gloria Hewitt

- Kenschaft, Patricia C. (1982, April). Black Women in Mathematics in the United States. Journal of African Civilizations, pp. 68-70.
- Fasanelli, Florence. "Gloria Conyers Hewitt," *Notable Women in Mathematics: A Biographical Dictionary,* Charlene Morrow and Teri Perl, Editors, Greenwood Press, 1998, 76-79.

Cleopatria Martinez

- Kenschaft, Patricia. *Change is possible*. American Mathematical Society, 2005, 121-124.
- Dr. Cleoapatria Martinez – Mathematician. SACNAS Biography Project. Available at http://www.sacnas.org/biography/Biography.asp?mem=65&type=2

Edna Lee Paisano

- Perl, T. *Women and Numbers: Lives of Mathematicians plus Discovery Activities*. World Wide Publishing/Tetra, 1993, 121-138.
- Sterrett, Andrew. 101 Careers in Mathematics. The Mathematical Association of America. Washington, D.C. 1996, 136.

Jean Taylor

- Albers, Donald J. "Still Questioning Authority: An Interview with Jean Taylor," The College Mathematics Journal, Vol 27, No. 4 (September 1996), 250-266.
- Kanigel, Robert. "Bubble, Bubble: Jean Taylor and the Mathematics of Minimal Surfaces," The Scientist, May/June 1993, reprinted in Math Horizons, Mathematical Association of America, September 1994, 5-11.
- Ruopp, Faye Nisonoff. "Jean Taylor," in *Notable Women in Mathematics: A Biographical Dictionary*, Charlene Marrow and Teri Perl, Editors, Greenwood Press (1998), p.252-257.
- What's Happening in the Mathematical Sciences, Volume 1, 1993, American Mathematical Society, 31-34.
- Peterson, Ivars. The Mathematical Tourist, W. H. Freeman and Company, 1988, p.61-70.
- Jean Taylor, "In Her Own Words," AMS Notices, Vol. 38, No. 7, Sept. 1991, pp. 702-706. (Reprinted at AWM web site)

Sylvia Wiegand

- Sullivan, Kathleen. "Sylvia Young Wiegand," *Notable Women in Mathematics: A Biographical Dictionary,* Charlene Morrow and Teri Perl, Editors, Greenwood Press, 1998, 272-277.
- Rowan, Karen. "It Adds Up to Success: Warming the Climate for Women in Mathematics," On Campus with Women, Association of American Colleges and Universities, Spring/Summer 2003 Newsletter.

Rubric for Contemporary Woman Mathematician Poster & Presentation

Be sure your poster includes a title with Contemporary Woman Mathematician's Name and the Poster Author's Name and each of the following sections (Possible point count for each section or requirement detailed below is noted in parentheses):

- Early Life and Education (5)
- Attraction to Mathematics (5)
- Career Opportunities and Choices (5)
- Obstacles She Faced & Overcame (10)
- Compare/Contrast to Earlier Women (15)
- Recognitions/Honors/Firsts (5)
- Modern Themes (Obtaining a job; Child care, if appropriate) (10)
- SOMETHING SPECIAL (For example, Her Reflections on Mathematics) (10)
- Bibliography (include photo sources, on the sample poster these are noted under each photo as well) (5)

Additional points for following directions, submitting on-time a poster with error-free writing and overall good appearance (10)

Your 10-minute in-class presentation will be assessed based on your overall preparation, including effective presentation, ability to field questions, making eye contact, little reliance on notes, appropriate professional dress (20)

Total of 100 points is possible.

III.6.4 Lesson Plan Assignment for Future Teachers

Future teachers who enroll in the course are encouraged to self-identify when it comes to making a choice between completing a poster on a contemporary woman mathematician or developing a lesson plan that connects a mathematical topic to a woman mathematician. This choice is made freely by the students in the last month of the semester. By that time many future teachers have already chosen to pursue one of the research projects that was indicated as suitable for future teachers. The lesson plan assignment was developed to address the additional learning outcome (Section II.1) specifically addressed to future teachers:

> Future Teachers will be able to
>
> P1. Prepare and share mathematics teaching resources and lesson plans (including appropriate assessments) that reflect equity principles

Following are the directions and rubric for the lesson plan assignment and a bibliography of resources for the lesson plan topics.

Directions and Rubric for Lesson Plan Assignment for Future Teachers

When writing your lesson plan include each of the following features in a clear way. The possible points assigned to each item is indicated in parentheses. The total possible is 100 points.

1. Assumed PREREQUISITE knowledge - mathematical and otherwise (5)
2. Lesson OBJECTIVES (instructional goals) - mathematical and otherwise (5)
3. LIST (with definitions) any special mathematical terms encountered in the lesson (5)
4. LIST OF MATERIALS needed, with sources cited (5)
5. BRIEF "BIOGRAPHY" of the person/organization that is connected to your lesson (5)
6. A description of how the lesson will proceed that includes activities and estimated times. Typically, you will describe how you plan to:
    a. get INTO the lesson by starting with a "hook" to catch interest,
    b. teach THROUGH the lesson by providing specific directions for the teacher and the students, and
    c. go BEYOND the lesson with an appropriate summary, assignment, and possible extension (20)
7. Address CLASSROOM MANAGEMENT (for example, how will materials be distributed/collected, or how will students be put into pairs or groups?) (20)
8. An ASSESSMENT plan that will tell you if your students achieved your instructional goals. PLEASE READ the two references* provided about assessment *before* your finalize your assessment plan. (5)

9. Possible FOLLOW-UP or related assignments (perhaps in other subject areas) (5)
10. ADVICE/REFERENCES to anyone who might try to adopt your lesson plan (5)

Your 10-minute in-class presentation will be assessed based on your overall preparation, including effective presentation, ability to field questions, making eye contact, little reliance on notes, appropriate professional dress (20)

*Assessment References

"An Assessment Model for the Mathematics Classroom," *Mathematics Teaching in the Middle School*, v.6, no.3, November 2000, 192-194.

"Gaining Insight into Students Thinking Through Assessment Tasks, *Mathematics Teaching in the Middle School*, v.6, no.2, October 2000, 136-144.

A Bibliography of Resources for Lesson Plans

1. Parker, Marla (ed.) (1995) *She does Math! Real life problems from women on the job.* Washington DC: Mathematical Association of America.
2. Perl, Teri (1993) *Women and Numbers: Lives of women mathematicians plus discovery activities.* San Carlos, CA: Wide World Publishing/Tetra.
3. Warren, Rebecca and Thompson, Mary. (1994) *The Scientist within You: Experiments and biographies of distinguished women in science*. Eugene OR: ACI Publishing.

For Claudia Zaslavsky, see [1] pp. 152-155 and the following:

Zaslavsky, Claudia (1973) *Africa counts; number and pattern in African culture.* Boston: Prindle, Weber & Schmidt.
--- (1996). *The multicultural math classroom: bringing in the world.* Portsmouth, NH: Heinemann.
--- (1993). *Multicultural mathematics: interdisciplinary, cooperative-learning activities.* Portland, Maine: J. Weston Walch.
Wiest, Lynda. (2002, September). Multicultural Mathematics Instruction: Approaches and Resources *Teaching children mathematics*, 49-55.

For Jean E. Taylor, see [1] pp 155-160 and the following:

Albers, Donald J. (1996). Still Questioning Authority: An Interview with Jean Taylor *College Math Journal, 27*(4), 250-266.
Kanigel, Robert. (1994, September). Bubble, Bubble: Jean Taylor and the mathematics of minimal surfaces. *Math Horizons,* 5-11.

For EQUALS and Math for Girls founders/authors, see [2] pp. 169-191 and the following:

Afflack, Ruth. (1982). Beyond EQUALS. Mills College, Oakland CA: Math/Science Network
Downie, D., Slesnick, T., and Stenmark, J. (1981). Math for girls and other problem solvers EQUALS: Lawrence Hall of Science Berkeley.

For Evelyn Boyd Granville, see [2] pp. 94-107.

For Emmy Noether, see [2] pp. 61-75 and [3] pp. 46-53.

For Sonya Kovaleskaya, see [3] pp. 144-151.

For Edna Lee Paisano, see [2] pp. 120-137.

For Florence Nightingale, see the following:

Franklin, Christine. (2002, February). The other life of Florence Nightingale Mathematics Teaching in the Middle School, 337-399.
Johnson, Art. (1999, March). Now and then: displaying data is as easy as pie! Mathematics Teaching in the Middle School, 391-397.

For Theoni Pappas, see [2] 155-167 and the following:

Pappas, Theoni. (1997). The Adventures of Penrose the Mathematical Cat. San Carlos, CA: Wide World Publishing/Tetra.
Pappas, Theoni. (1989). The Joy of Mathematics. San Carlos, CA: Wide World Publishing/Tetra.
Pappas, Theoni. Children's Mathematical Calendar published annually.

On STEM Career Participation by Women

Gender Differences at Critical Transitions in the Careers of Science, Engineering, and Mathematics Faculty by Committee on Gender Differences in the Careers of Science, Engineering, and Mathematics Faculty; Committee on Women in Science, Engineering, and Medicine; Committee on National Statistics; National Research Council, 2010. Available online at http://www.nap.edu/catalog.php?record_id=12062#toc

III.6.5 Final Reflective Writing Assignment

As indicated in Section II.3, there is no final exam for the course. Instead, an end-of-term reflective writing assignment concerns itself with the following critical learning outcomes:

W2. Synthesize from these women's biographies common experiences/obstacles faced by women who wished to participate in mathematics and identify factors that enabled their success
G2. Read critically articles in journals and newspapers dealing with gender issues in mathematics or science education
M4. Recognize and appreciate: mathematics as a study of patterns; the critical and distinct roles of inductive and deductive reasoning in developing (new) mathematics; the utility of multiple representations for a single mathematical concept

The student work resulting from this assignment can also provide instructors with insights into student understanding and can direct revisions to improve the course. The directions for this assignment follow.

Final Reflective Writing Assignment

There is no final exam in this course. Instead you are asked to take time to review and reflect on the major areas of concern in this course. Specifically, this assignment asks to you to write on three topics, and to submit one to two pages typed, double-spaced, Times New Roman, font size 12, for each topic. We are looking for thoughtful, well-organized responses to the prompts for each of the topics. NOTE: These are reflection and synthesis pieces; there is no expectation for you to provide supporting references or formal citations, although you are welcome to do so informally.

Topic 1: For the 9 historical women mathematicians studied in this course, what were some shared experiences and obstacles for these women in their pursuit of mathematics. Identify the commonly occurring factors that enabled their success in overcoming these obstacles. How have things have changed for modern women mathematicians (say, for women in the mathematical sciences in the last 20 to 30 years)? As indicated by your readings, what challenges still remain for women in mathematics in the 21st century?

Topic 2: We recently returned to the questions posed at the beginning of the semester about what mathematics is and how new mathematical knowledge is produced. Attached to this sheet you have a copy of your responses from the beginning and middle of the semester. Now we would like you to give your current view. Describe how the course content, activities or experiences (readings, mathematical assignments and discussions, projects, poster or lesson plan) have influenced your ideas (either *changed* or *reinforced* them) about these two questions. Include *at least* one specific example or reference to course content or activities or experiences.

Topic 3: The semester is over, and you have some free time. You are surfing the web and come across ****MenRSmartR@blogspot.com***** where you find the remark: "In all of history there have been no women geniuses and this clearly indicates males have a superior intelligence!" Draw on material from this course to craft a response (350 word limit to all responses on this blog).

III.7 Additional Bibliography

Sections III.6.1 through III.6.4 contain additional bibliographical references for the reports, research projects, posters and lesson plan assignments, respectively. This section contains additional references listed by category.

Basic resources appropriate for K-12 students
about the 9 Women Mathematicians from *Math Equals*

(J = suitable for junior high school; S = suitable for senior high school)

Campbell, P. and L. Grinstein. *Women of Mathematics: A Biobibliographic Sourcebook*, New York: Greenwood Press, 1987 (S)

Downie, D., Slesnick, T. and J. Stenmark. *Math for Girls and Other Problem Solvers,* Berkeley, California: Lawrence Hall of Science, 1981 (J)

Kenschaft, P. "Black Women in Mathematics in the United States," *The American Mathematical Monthly*, October, 1981 (S)

Kovalesky, S. *A Russian Childhood*, New York: Springer-Verlag, 1978 (S)

Osen, L. *Women in Mathematics*, Cambridge: MIT Press, 1974 (S)

Perl, T. *Math Equals*, Menlo Park, California: Addison-Wesley, 1978 (S)

Perl, T. and J. Manning. *Women, Numbers and Dreams*, Windsor, California: National Women's History Project, 1981 (J)

Reimer, L. and W. Reimer. *Mathematicians Are People, Too,* Palo Alto, California: Dale Seymour Publications, 1990 (J)

More Publications Concerning Gender Differences in Mathematics Performance

Hyde, J., Lindberg, S., Linn, M., Ellis, A., & C. Williams, C. (2008). Gender similarities characterize math performance, *Science* 321, 494–495.

Hyde, J. and Mertz, J. (2009). Gender, culture, and mathematics performance. *Proceedings of the National Academy of Sciences, 106*(22), 8801–8807.

Kane, J. & Mertz, J. (2012). Debunking Myths about Gender and Mathematics Performance. *Notices of the Amer. Math. Soc. 59*(1), 10-21.

Lindberg, S., Hyde, J., Petersen, J., & Linn, M. (2010). New trends in gender and mathematics performance: A meta-analysis, *Psychol. Bull.* 136, 1123–1135.

On STEM Career Participation by Women

Committee on Gender Differences in the Careers of Science, Engineering, and Mathematics Faculty; Committee on Women in Science, Engineering, and Medicine; Committee on National Statistics. (2010). *Gender Differences at Critical Transitions in the Careers of Science, Engineering, and Mathematics Faculty*. National Research Council. Available online at http://www.nap.edu/catalog.php?record_id=12062#toc

IV. Tensor-MAA Grant Report, Research and Dissemination Links

1. Final Report for the "Women and Mathematics for Future Teachers" Tensor-MAA (2007-2011)
2. Research Related to this Course and funded in part by Tensor-MAA (2007-2011):
    - 2.1. "Future Teachers' Views of Mathematics and Intentions for Gender Equity: Are These Carried Forward into Their Own Classrooms?", preliminary report by J. Dewar presented at 15th annual conference, Special Interest Group of the MAA on Research in Undergraduate Mathematics, Portland, OR, February 25, 2012.
    - 2.2. "Using the History of Women in Mathematics to Address Gender Equity and Prepare Future Teachers," paper by J. Dewar, A. Crans, and L. Khadjavi, Contributed paper session on the history and philosophy of mathematics, Joint Mathematics Meetings, Boston, MA, January 7, 2012.
        - 2.2.1. PowerPoint Slides
        - 2.2.2. Handout
    - 2.3. "Women & Mathematics" for Future Teachers: Outcomes of a Tensor-MAA Women and Mathematics Grant, poster by A. Crans, J.Dewar and L. Khadjavi, Mathematics Outreach Poster Session, Joint Mathematics Meetings, Boston, MA, January 5, 2012.
        - 2.3.1. Poster
        - 2.3.2. Handout
    - 2.4. "Mathematics and Equity, Past and Present, through the Lives and Work of Women Mathematicians," paper by J. Dewar, A. Crans and L. Khadjavi, MAA Contributed Paper Session on Humanistic Mathematics, Joint Mathematics Meetings, New Orleans LA, January 8, 2011.
        - 2.4.1. PowerPoint Slides
        - 2.4.2. Handout
    - 2.5. What is Mathematics? A Scholarship of Teaching and Learning Investigation

IV.1 Final Report for the "Women and Mathematics for Future Teachers" Tensor-MAA (2007-2011)

Report on Round 1 Funding 2007 - 2010

In Spring semester 2008, Dr. Dewar team-taught with Dr. Khadjavi MATH 398 Women and Mathematics (cross-listed with honors and with women's studies). Of the 23 students enrolled, 10 were math or science majors/minors, 9 of whom definitely planned on a career in teaching. Thus, we far exceeded our goal of an enrollment of 12 students overall (that is 50% more than the previous offering of the course), and nearly reached our goal of 12 future math/science teachers.

Students who took MATH 398 in Spring 2008 have:

- Led hands-on workshops involving mathematical activities and information about women mathematicians at the 2008 and 2009 Expanding Your Horizons (EYH) Career Days to a total of 75 middle/junior high school girls
- Made presentations to 65 potential future teachers (30 attendees at the Future Teachers Conference in October 2008 and 35 attendees at the Pacific Coast Undergraduate Mathematics Conference (PCUMC) in April 2009)
- Agreed to lead workshops at the 2010 EYH and PCUMC conferences.

So after only one of the originally proposed two team-teachings of the course, we reached 75% of the original goal for outreach by course 'graduates' to young girls and 65% of the outreach by 'course graduates' to future teachers. After the 2010 conferences we expected to reach 100% of both outreach goals, just through the work of the first set of 'graduates.' Therefore in the renewal grant we proposed to achieve 150% of our original outreach goals after the second team-teaching.

A second round of funding was requested for the following reason. Drs. Khadjavi and Crans had been scheduled to team-teach the course in Fall 2009, but the economic crisis of 2009 and university budgets required Dr. Crans to teach a different course at the same time. Dr. Khadjavi taught MATH 398 Women and Mathematics by herself in Fall 2009 and 7 students were initially enrolled, 6 completed the course, all of whom were potential K-16 math/science teachers, 2 in K-8. With the second round of Tensor-MAA Dewar and Crans were able to team-teach In Fall 2010. Completion of the course manual and website and the dissemination of the project results at the Joint Mathematics Meetings, as proposed in the original grant, were deferred until the after the second team teaching.

Report on Round 2 Funding, 2010 - 2012

The 2010 renewal of the Women and Math for Future Teachers grant supported a second team teaching of the course Women and Mathematics (Dr. Jackie Dewar with Dr. Alissa Crans) at Loyola Marymount University (LMU) in Fall 2010. Eleven students completed the course, bringing the total number of enrollees during the grant period to 40. Outreach efforts, performed by former students in the course who were encouraged and mentored by the instructors (Crans, Dewar, and Khadjavi), continue to reach growing numbers of younger students and current and future K-12 teachers. As of February 1, 2012, total outreach achieved by LMU students, since the original grant funding in 2007, has been:

125 junior high girls attended EYH workshops led by LMU students
55 undergraduates attended Pacific Coast Undergraduate Mathematics Conference talks
70 teachers and future teachers attended Future Teachers Conference talks/workshops

Faculty presentations in MAA contributed paper sessions at the 2011 and 2012 Joint Math Meetings reached about 45 teachers/students and a poster in the 2012 MAA Outreach Session disseminated information to approximately 20 faculty/students. An upcoming talk on February 25, 2012 at the RUME conference will likely reach another 20. Another EYH workshop will be presented by LMU students on March 17, 2012 to 25 more junior high girls. Recent interviews with (just) three former students, now teaching in LA schools, revealed that these former students have been using information and materials from the course with their own students. This documents that another 165 secondary and 20 elementary students have learned that "interesting women have been doing interesting mathematics throughout history." Targeting future teachers in this grant has proven to be an excellent strategy for achieving significant and increasing impacts.

IV.2 Research and Dissemination related to this Course and funded in part by Tensor-MAA (2007-2011)

IV.2.1 "Future Teachers' Views of Mathematics and Intentions for Gender Equity: Are These Carried Forward into Their Own Classrooms?" preliminary report presented by J. Dewar at 15th annual conference, Special Interest Group of the MAA on Research in Undergraduate Mathematics, Portland, OR, February 25, 2012.

See http://myweb.lmu.edu/jdewar/wam/files/RUME15.pdf

IV.2.2 "Using the History of Women in Mathematics to Address Gender Equity and Prepare Future Teachers," paper by J. Dewar, A. Crans, and L. Khadjavi, Contributed paper session on the history and philosophy of mathematics, Joint Mathematics Meetings, Boston, MA, January 7, 2012.

2.2.1 PowerPoint slides

See http://myweb.lmu.edu/jdewar/wam/files/JMM2012ppt.pdf

2.2.2 Handout

See http://myweb.lmu.edu/jdewar/wam/files/JMM2012handout.pdf

IV.2.3 "Women & Mathematics" for Future Teachers: Outcomes of a Tensor-MAA Women and Mathematics Grant, poster by A. Crans, J.Dewar and L. Khadjavi, Mathematics Outreach Poster Session, Joint Mathematics Meetings, Boston, MA, January 5, 2012.

2.3.1 Poster

See http://myweb.lmu.edu/jdewar/wam/files/JMM2012poster.pdf

2.3.2 Handout

See http://myweb.lmu.edu/jdewar/wam/files/JMM2012posterhandout.pdf

IV.2.4 "Mathematics and Equity, Past and Present, through the Lives and Work of Women Mathematicians," paper by J. Dewar, A. Crans and L. Khadjavi, MAA Contributed Paper Session on Humanistic Mathematics, Joint Mathematics Meetings, New Orleans LA, January 8, 2011.

2.4.1PowerPoint slides

See http://myweb.lmu.edu/jdewar/wam/files/JMM2011ppt.pdf

2.4.2 Handout

See http://myweb.lmu.edu/jdewar/wam/files/JMM2011handout.pdf

IV.2.5 What is Mathematics? A Scholarship of Teaching and Learning Investigation

See http://www.cfkeep.org/html/snapshot.php?id=30198362